# Interaction 1: Session 1 (9:40 – 10:35)

Sean: What are you thinking?

Uma: [Looking at her paper] So you have to figure out (5 sec) I feel like these ones [pointing to dates on the problem sheet] would be… Is it like… Represent the following dates on a number line [reading the task] (4 sec)

Sean: So what I’m seeing right off the bat is that these numbers [pointing to Uma’s problem sheet] are so big that if we make it just to one scale these are all going to be clumped up at the end like super close to each other

Uma: Yeah, they’re going to be like right there, and then these ones are going to be more—Is it already in order?

Sean: I think it is [now looking at his own paper]

Uma: Ok, because that’s what it looks like. Because it looks like the numbers are getting bigger.

Sean: M hm [affirming]

*The interaction ends with both students continuing to read the problem sheet silently.*

Building Significance?

To begin this interaction, Sean invites Uma to share her thinking about the task. It is unclear whether Sean is inviting her mathematical ideas, interpretation of the task, or just her thinking broadly. Uma thinks aloud while reading from the task sheet, suggesting that she is still thinking about the task and what is being asked of them. After minimal wait time, Sean then presents an argument for why he thinks using a single scale for their timeline would be problematic, which Uma revoices and agrees with. By revoicing Sean’s argument, Uma is demonstrating that she understands his mathematical thinking and positions his thinking as mathematically significant. The interaction concludes with Uma presenting an argument about the arrangement of the dates listed on the task sheet. Sean agrees with Uma’s argument that the dates are given in order without attributing mathematical significance to the observation.

Building Communal Practices?

In this interaction, Sean originally invites Uma to share her thinking about the task, where later talk-turns suggest he already had a direction in mind. When Sean presents his argument for why clumping of smaller values might take place, Uma responds by revoicing his thinking and agreeing with it. In doing so, Uma demonstrates that she is taking up Sean’s idea and values it. Then, Uma presents a claim in the form of a question (this is a claim because she backs it up with evidence about the origins of her question), which can be interpreted as an invitation for Sean to consider her mathematical observation. Sean answers her question with a statement that ultimately ends the interaction, and begins working on his own paper to carry out his approach to creating a timeline.

Interaction 2: Session 1 (11:08 – 14:40)

Uma: Where would you want to start?

Sean: (4 sec) Well I’m kind of thinking um, that dividing it into kind of eras (4 sec) and then kind of having each era have its own scale kind of so we can look at the line and you know if you’re back three eras then within that era we can tell where things are at um…

Uma: M hm, yeah [affirming]

[44 sec – Sean is using a straightedge to begin his timeline and working on his own paper. Uma is using a straightedge and regularly looks to Sean and his paper]

Uma: So did you want to start with “now” up here?

Sean: Yeah, I think so. Most current on the right—

Uma: So like right here is “now”—

Sean: Yep

Uma: --and like all the way back here is where you have this last one.

Sean: M hm [affirming]. And then like I say, one way we can alleviate that clumping is by—can we write on this [whole group laughter]— So maybe like the first epoch or whatever is to there [pointing on his own paper], you know, and I’m just like—

Uma: You’re just kind of picking

Sean: Yeah, when we get into this we might realize like that’s not gonna—

Uma: Yeah [nodding] we may have to re-evaluate

Sean: Yeah, so I’ll go—[3 sec. writing on his paper]— That’s what I’m gonna do for now [inaudible] any out there, but I think we’re going to wanna shift it that way.

Uma: M hm [affirming], yeah probably.

Sean: So we’ve got. So kind of my thought is we represent em all um—so ignore that mark [marking on his paper]—is that we represent each era as an equal—

Uma:--Amount of time

Sean: Yeah—

Uma: Or equal space/

Sean: /[concurrently with Uma] equal distance.

Sean: Yep, um. And then we can kind of figure out you know this is 10 thousand and then this is 6.6x107 [getting quieter, and working on his own paper], and [trails off while working on his own paper]

Uma: How did you know how far? Did you just kind of space them out?

Sean: Well what I did was I just kind of like as far as dividing it into fourths I just eyeballed that

Uma: M hm, yeah

Sean: But then the last date in that—

Uma: --in that category

Sean: Yeah, and then the last date there, and the last date there, and then… [pointing to different dates on the problem sheet, and then continuing to work on his paper]

Sean: (7 sec) And don’t let me lead you astray.

Uma: Haha, yeah

Sean: You know*,* if you have a thought, pleeeeease

*The interaction ends with an 11 second pause, where Sean is visible on camera implementing his strategy before Uma changes the mathematical topic by arguing (in the form of a question) that converting all of the dates on the problem to scientific notation might be beneficial, which is the start of a new interaction.*

Building Significance?

This interaction begins with Uma inviting Sean to collaborate on solving the problem. In her invitation, she establishes Sean’s mathematical thinking as [potentially] significant in their work towards a solution. Throughout the interaction, Sean frequently states his thinking about an approach to segmenting the timeline into eras to prevent “clumping.” During Sean’s statements, Uma regularly affirms his suggestions and demonstrates that she understands his thought process by finishing his sentences with mathematically relevant information. She continues to position Sean’s mathematical thinking as significant by implementing his approach on her own paper. Sean recognizes that his thinking is being privileged and attempts to reconcile in his last talk-turn of the interaction. Overall, Uma positions Sean’s mathematical thinking as significant regularly and throughout the interaction. There are no instances of Sean positioning Uma’s thinking, mathematical or otherwise, as significant in this interaction.

Building Communal Practices?

Regarding the building task of inviting others into the mathematical practices, Uma initiates the interaction with an invitation about mathematical solutions and then revoices, agrees, and implements the strategy suggested by Sean. On the other hand, Sean makes statements. Sean does not provide any data or justification to his statements, and routinely refers to what “I” will do to execute the solution strategy.

# Interaction 3: Session 1 (14:45 – 16:00)

Uma: Would it help to change these… so that they’re all in the same?

Sean: So write it like, 5 times 10—squared [concurrent with Uma below]

Uma: --to the second. Yeah.

Sean: 1 times 10 to the third [apparently saying what he is writing, not particularly talking to Uma]. 1 point 8 [trailing off. Getting quitter]

Uma: 1800 is that—

Sean: I put 1.8 times 10 to the third.

Uma: Ok, to the third.

Sean: I think so [nodding. Looking at his paper]

Uma: I was like, I couldn’t tell if it was like to the third or to the—

Sean: [nodding. Looking at his paper] It would be 18—

Uma: Yeah because you have to move it over three. Yeah.

Sean: Yep. You got her.

Uma: It’s been a while since I’ve done this [laughing]

Sean: Yep. That’s part of the fun, right. [looking at his paper].

Uma: M hm [affirming]. [Whispering, seemingly to herself]1 times 1, 2, 3, 4, times 10. 1, 2, 3, 4, 5 times 10

*The camera is not positioned to capture Uma’s body language, gestures, or gaze during this interaction. Sean is seen to be writing on his own paper throughout. The interaction ends with both students converting all values on the task sheet to scientific notation.*

Building Significance?

Uma begins this interaction by inviting Sean to consider a mathematical idea (converting all dates on the task to scientific notation) through her initial question. Sean first clarifies that Uma was intending to write all dates in scientific notation, and in doing so cut her off from finishing her initial question. Sean then proceeds to implement Uma’s suggestion without providing her with any cues, such as revoicing, about whether he valued her idea as mathematically significant. On the one hand, by implementing her idea, Sean could be implying that he views Uma’s contribution as mathematically significant towards completing the task. On the other hand, Sean continues working on his own paper, never looks to Uma, nor does he participate in a conversation around her idea. Instead, he provides short responses to Uma’s specific questions, and appears to be solely focused on his own work.

Building Communal Practices?

Uma’s original questions served as an invitation for Sean to interact with her around a mathematical idea. Sean’s clarifying question and efforts towards implementing her idea (on his own) suggest he agreed with her thinking, although he did not appear interested in further discussion. Uma makes a second attempt at conversing with Sean through her question about how to convert one of the values to scientific notation, which a later talk-turn suggests she did not need to ask. Her statement, “because you have to move it over three” implies that she knew how to convert the value to scientific notation, and could suggest that she was again seeking to continue talking with Sean. In this sense, Uma may have been sacrificing her appearance of seeming mathematically competent to Sean in order to promote social norms within the group.

# Interaction 4: Session 1 (16:15– 18:05)

DW: [Approaching from over Sean’s right shoulder, opposite from Uma. Pointing to Sean’s paper] Can you- can you say that to me, and then talk to me about what you guys are thinking?

Sean: Sure. So kind of what it is that we’re talking about is if we put all of this on one line of uniform scale then we are going to end up with these numbers up here scrunched down to one end [gesturing ‘scrunched’ with both hands]. So we are kind of dividing it- attempting to divide it into eras, and each era will kinda have its own scale but they’ll be on the same line still to give you the impression of distance back.  
  
[Through this talk turn, Sean is talking directly to DW. Uma is looking on her own paper, occasionally nodding with what Sean states.]

Sean: [turning to Uma] Does that sound about right to you? Does that kind of…?

Uma: M hm, yeah.

DW: Uma, can you- Uma can you explain more about what he meant by the numbers up here [pointing to Sean’s paper] are going to be squished?

Uma: Um, just kind of that these numbers are so much smaller, then the further you get out here the bigger this number is gonna be [pointing to her paper]. So those ones are all closer together because their numbers are closer together, so they are going to be closer.

DW: Closer…?

Uma: As in like the same size.

DW: Relative to what? You seem to be saying that as a comparison?

Uma: Like… um… Kind of just like, relative to like the number. So like these numbers are closer together [pointing to the smaller values on the task sheet] and then these numbers are farther apart because they’re bigger numbers. Kind of thing.

DW: Ok, alright, alright [walking behind Sean and reposition on Uma’s right shoulder]. And then you said you’re chunking into different eras?

Sean: Yeah.

DW: And then you’re going to work within those different chunks?

Sean: Yeah, yeah [both Uma and Sean are nodding in agreement].

The interaction ends when DW walks away from the conversation.

Building Significance?

In this interaction, DW does much to position Sean’s thinking as significant while simultaneously diminishing Uma’s thinking. First, DW prompts Sean (use of “you” and pointing to Sean’s paper) to share about “you guys’” thinking, which positions Sean’s thinking as significant. Then, DW asks Uma to further explain “what HE meant” once again positioning Sean’s thinking as mathematically significant. Following Uma’s explanation and restatement of the argument presented by Sean, DW does not interact with her thinking beyond “Ok, alright, alright.” Finally, to close the interaction, DW revoices Sean’s original approach to establishing eras for solving the task, which again positions his mathematical thinking as significant. Further, DW’s talk-turn may also have worked to diminish Uma’s mathematical thinking and establish a mathematical hierarchy between Sean and Uma.

Based on analyzing the interactions preceding this one, we also know that Sean neglected to share Uma’s mathematical idea of converting all dates to scientific notation (which becomes important later when the pair decide that eras based on powers of 10 help alleviate the clumping of events on their timeline) when explaining “their” approach to the task. Instead, Sean only discussed the mathematical idea he authored in the first interaction.

Taken together, DW’s actions and language within this interaction along with Sean’s choice to share only his mathematical idea reinforce the hierarchy being established between Sean and Uma.

Building Communal Practices?

In what ways do the partners’ language and actions invite others into the communal mathematical practices?

Interaction 5: Session 1 (18:20 – 18:55)

Sean: [Pointing to his work. Uma is leaning over to see.] So even on this first like scale I’m seeing, like it’s still going to be pretty clumped because I’ve got like a thousands there, 500, 1800, 2000, so even at this one—

Uma: --They’re still going to be really close. There’s kind of no way around, they’re close together kind of [Stops looking at Sean’s paper and looks to the group on the opposite side of the table].  
  
[Uma pauses for 10 seconds, looking towards Sean and his paper, before beginning to work on her own paper. Sean continues working on his paper without looking up.]

*The interaction ends with 3 minutes and 43 seconds of Sean and Uma working on their own papers to implement the “eras” strategy before DW re-enters to explain that he will eventually ask them (pointing to Sean’s paper) to explain the spacing of dates within their eras.*

Building Significance?

Uma positions Sean’s thinking as significant by revoicing and agreeing with his argument for why the dates within their eras will still be clumped together.

Building Communal Practices?

Sean invites Uma to interact with his thinking by leaning towards her and showing her the issue that HE is encountering. Sean does not respond to Uma’s claim that “there’s kind of no way around [it]” and instead continues to work on his own paper. Uma appears to be waiting for Sean to respond by waiting 10 seconds before going back to her own paper. Note also that Sean states, “I’ve got like thousands there,…” which suggests that Sean viewed his work as HIS and not THEIRS.

Interaction 6: Session 1 (22:20 – 24:35)

DW: So I am, eventually, you guys can think about it for now, but I am curious to know how you decided how many eras to include. And then how you, how you are working within in those. [Standing between Sean and Uma. Pointing to Sean’s paper.]

Sean: Do you want to know that now?

DW: Yeah

Sean: Because [inaudible]

Uma: [laughing]

DW: I just want to make sure that you’re hashing that out together or something. I was just—Like that’s the natural question that I had after looking at your—

Sean: So from my perspective it was an arbitrary choice [DW laughs] with room to adjust if it didn’t kind of meet our needs.

DW: Okay. [pointing to Uma]

Uma: Yeah.

DW: What needs did you—What do you—What do you think or anticipate might force an adjustment?

Sean: So that [looking at DW, and then shifting his gaze to Uma]—do you [talking to Uma] wanna go or?

Uma: Um I mean it’s just kinda gonna come down to like how you’re gonna fit them all within each of the like chunks that we have. How you’re gonna get em, like how far apart and stuff they’re gonna be within the chunk [looks at Sean, who is nodding in agreement].

DW: Okay and how are you—how are you thinking about that within-chunk measure?

Sean: So, I guess what I would, maybe this will answer the question, but for this first era we put 10,000 as our threshold and we’re seeing that most everything is still pretty clumped up. So if we moved our threshold up to 2,000 we would see that distributed out, and maybe we could make a 5th era. But within those as far as unit of measure it’s just a linear distribution of dates within the era.

DW: Okay. Of the dates or of the number of dates? Because I want to make sure I understand. So within each era are you evenly distributing the dates or are you evening distributing the number of events on the sheet that go in that space?

Sean: So I don’t wanna, I don’t wanna [looking to Uma and gesturing towards her]

DW: That’s fine. I’ll let you guys talk through it.

*DW leaves the scene, which ends the interaction. Sean and Uma continue to talk about the questions DW posed, so the next interaction is an immediate continuation of the previous discussion.*

Building Significance?

DW’s frequent invitations for the pair to interact with him based on their mathematical work positions the group solution strategy and use of “eras” as significant. The mathematical ideas being discussed were authored by Sean in previous interactions, so his thinking is being positioned as significant.

Neither Sean nor Uma position either’s thinking as significant in this interaction.

Building Communal Practices?

DW’s invitations also serve as a method for building communal practices of collaborating on mathematics and forming a pair-generated solution to the task.

Interaction 7: Session 1 (24:35 – 26:10)

Sean: I don’t want to dominate the discussion so…

Uma: Yeah, no—I mean he [DW] brought up a good—because with this one because if you focus on like evenly distributing like the events on here, that’s gonna effect the spaces on your um thing. Because like if you’re to even like, you know, so that each chunk has the same amount of events that doesn’t mean that their distance is gonna be the same. So you’re still—you could still possibly get way more over here within your chunk and like nothing on that side. But it’s kinda hard to look at it as like the year because then it’s hard to—

Sean: Well that’s where— You know, maybe we would be better off with a 5th if we went with everything to the 9th as one era. Or maybe—

Uma: 9 and up, yeah.

Sean: Or maybe like 8th because then we’re gonna get a nice like 1.42 to—

Uma: That’s true, yeah. Because then those are gonna be the same. And then everything from the 7th –

Sean: And then maybe we go, I don’t know maybe somehow up here we do a couple eras within that even.

Uma: M Hm. Yeah because then you’re looking more at the numbers [referring to the dates for each event].

Sean: So there’s no shame in starting a new line [laughing].

Uma: Yeah. That’s what there’s the whole paper for.

Sean: Yep. There’s extra paper there too, so.

*Uma and Sean continue by creating a new number line that uses their new approach to creating eras, based on the dates of events as opposed to the number of events. Approximately 30 seconds go by before the next interaction. In that time, Uma can be heard thinking aloud and then she asks Sean a question.*

Building Significance?

This interaction appears to demonstrate a back and forth between Uma and Sean where there is mutual significance ascribed to each other’s thinking. Uma’s initial argument for why creating eras based on the number of events within each is inappropriate demonstrates her understanding of their original approach to the problem. Then, Sean builds from Uma’s argument to suggest making different eras based on the values associated with events on the task sheet. Sean only provides statements, which are closed (not inviting). On the other hand, Uma frequently revoices and accepts Sean’s statements to move forward with a new approach to sub-dividing the timeline. Uma’s revoicing (and completing Sean’s sentences) demonstrates that she is understanding the approach.

Building Communal Practices?

While this interaction demonstrates a mutual back and forth between Uma and Sean, there were still differences in the way each partner’s language was used to promote collaboration. Specifically, Uma provides an argument (i.e. claim, with evidence and reasoning) and then regularly revoices Sean’s statements. Presenting an argument is a way of inviting others to interact with one’s reasoning, and revoicing with accepting demonstrates a desire to reach consensus on another’s ideas. Sean, provides statements, which are relatively closed ways of interacting. Statements without reasoning do not invite others to interact with one’s mathematical thinking.

An interesting action at the end of the interaction is that both students proceeded to create new timelines individually. Then while creating the new timelines, Uma can be heard thinking aloud, which is not acknowledged by Sean.

Interaction 8: Session 1 (26:50 – 27:15)

Uma: So how many does that change it into si[x]? [looking at Sean’s paper]

Sean: Well what I did was, I just used a ruler here and said ‘well about 4 cm’ and just went with that.

Uma: Okay [picking up her ruler to implement on her paper]

Sean: and you’ll see that it’s not exactly 4 [cm] but it’s pretty close, so…

Uma: Yeah getting it as close as…[working on her own paper]

Sean: It just allows us to divide it up.

*The two work independently on their own paper for approximately 30 seconds. Uma can be heard thinking aloud.*

Building Significance?

Neither partner establishes the other’s thinking as significant in this interaction. This is a relatively neutral interaction.

Building Communal Practices?

Uma asks a question, which is an open form of reaching out to promote collaboration or discussion. Sean makes statements, which is a closed form of collaborating. Once again, Uma is heard thinking aloud while the two continue working independently. During which time, Sean does not acknowledge Uma’s thinking (or even that she is talking at all).

Interaction 9: Session 1 (28:03– 31:00)

Sean: So I kinda ran out of room here, but uh I just went 10 to the 10th, 10 to the 9th, 10 to the 8th [pointing to the different eras on his sheet. Turning towards Uma] and then will fit those within there.

Uma: Oh, ok! And then we’ll fit them. Yeah.

Sean: Yeah.

*The two work independently on their own papers [5 seconds]. Both students can be heard thinking aloud. In doing so, Uma turns to look at Sean’s paper and Sean states what he is about to do.*

Uma: [turning to look at Sean’s paper. Thinking aloud] 10 to the 6th.

Sean: Yep, 10 to the 5th, and then I’m going to start my line. Over here, I’m actually going to extend it on—[continuing to work on his paper]

Uma: [After working independently 9 seconds. Leaning towards Sean to look on his paper] And then you’re adding a little bit more? [returning to her own paper]

Sean: Yeah [continuing to work on his paper]

Uma: [5 sec.] Keeping the similar—

Sean: M hm [affirming]

Sean: [7 sec] Well, and that’s turning into a scale that I recognize [bobbing, smiling slightly, looking over his glasses to Uma].

Uma: Yeah, so then that’s… it goes to 10 to the 4th

Sean: M hm.

Uma: And then that’s going to be 10 to the 3rd, and then 10 to the 2nd [turning to look at Sean]?

Sean: Yep. And then I just went on out to—

Uma: 10 to the 0, yeah to make it easier [4 sec]. So then, “now” falls in at the zero mark, right? Right? Would that be where we have now?—

Sean: That’s now. Yep [working on his own paper]

Uma: [5 sec] And then 10 to the 2nd is 100, right.?

Sean: Yeah, so you want to kinda like—[continuing to work on his own paper]

Uma: So then you have—[working on her own paper]

Sean: [4 sec] So if you wanna like—[continuing to work on his own paper]. [7 sec] So if we—the difference from here is actually 900 years, right.

Uma: Yeah, but then with each one the years are going to be more [gesturing ‘expanding’ with her hands].

Sean: Yep. Yep. [nodding]

Uma: Yeah, that’s what I was thinking. I was like—because 500 is going to be because this is 1000, so your 500 is kinda going to be like half-way right.

Sean: Yeah [seems to have a hesitation in his tone. Watches Uma work on her paper for a brief moment before returning to his paper]

Uma: It should be like here, 500. And then this will be 1000 [working on her own paper].

Uma: [6 sec, thinking aloud] 10 to the 4th right here.

*The interaction ends with about 54 seconds of Sean and Uma working independently on their own papers. Uma occasionally thinks aloud. Neither student acknowledges the other in this time period. The next interaction begins when DW walks behind/between the two to look onto their papers. Sean asks Uma, “What are you thinking?”*

Building Significance?

This interaction begins with Sean stating how he is creating eras on his timeline based on powers of 10. Uma picks up this idea, and demonstrates her approval and understanding of the approach through revoicing, agreeing, clarifying, and implementing the approach. Uma frequently asks clarifying questions while leaning towards Sean to look on his paper. She demonstrates understanding of the solution approach by stating where “Now” falls on the timeline (and also phrasing this as a question). Uma’s talk-turns throughout the interaction establish Sean’s approach as mathematically significant. Sean does not position Uma’s mathematical thinking as significant, even when Uma presents an argument for where 500 should be located on the timeline. In fact, from Sean’s hesitation in responding to Uma in the moment (and based on what follows in interaction 15), it seems as though Sean does not agree with Uma’s argument. Though in this interaction he does not question her thinking, nor does he elect to have a conversation about his thinking (that later shows up in interaction 15).

Building Communal Practices?

Uma regularly phrases her talk-turns as clarifying or general questions, even when she presents her argument for the placement of “Now” and 500. These uses of language promote a response from Sean, and are therefore open. Uma is trying to initiate conversation with Sean about her thinking and understanding of the solution strategy he authors.

On the other hand, Sean phrases almost all of his talk-turns as statements or very short acknowledgments. Although there is much back and forth in this interaction, Sean’s use of language appears to be rather closed.

Interaction 10: Session 1 (31:40 – 32:33)

*DW has been standing behind/between Sean and Uma for about 14 seconds, maneuvering to see what they are writing on their individual papers.*

Sean: What are you thinking [to Uma]?

Uma: Well I got out a different color so that I could mark the different—so like we have the different—the different parts so I can use a different color to mark the points that fall within these points on the line. But… I’m trying to figure out 5000… [quietly] 1800 is going to be— it’s going to be more over here, right. 1800 is going to be 800 more [getting quieter]

Sean: I think it’s going to be—it depends on how we—

Uma: How you look at the scale within the--

*The interaction changes to a new interaction because DW enters the conversation by asking a question.*

Building Significance?

This interaction does not feature either student positioning the other’s thinking as significant, perhaps with the exception of Uma’s last talk-turn where she finishes Sean’s sentence regarding the nature of the scales within their powers of 10.

Building Communal Practices?

Sean initially invites Uma to share her thinking about what she’s working on. After listening to Uma think aloud as she positions the event at 1800, Sean begins to state his thought about considering the scale within segments. Uma acknowledges his statement while working.

Interaction 11: Session 1 (32:32 – 33:51)

DW: So how did you? You guys seem to have moved from the original line to a new line. Can you tell me about the transition or what prompted it?

Uma: [Leaning back in her chair] Um, well after talking about like the difference between the like making it an even amount of the events we evaluated and maybe it would make more sense to make an even amount of time because some of the events—because if you were going to chunk it based on events—it just didn’t seem like the scale was gonna make much. So we figured if we looked more at like a number scale then we could plot the points on here within the number scale.

DW: Ok, so you’re still thinking of chunking it into different blocks, but how you’re working within those blocks is now based on amount of time from the events within as opposed to the number of events themselves?

Uma: M hm [affirming]

DW: Ok. [Leaning over to point to Uma’s paper] And you appear to be using the exponent on 10 to delineate where—

Uma: Where those points are. Yeah.

DW: Ok. Neat-o [walking away]

*The interaction changes to a new interaction because DW leaves the conversation.*

Building Significance?

Uma presents an argument for why they abandoned their previous approach and the affordances of the new approach as a response to DW’s initial question. Then, by revoicing Uma’s language and posing clarifying questions (also pointing to Uma’s paper), DW is positioning Uma’s response as mathematically significant.

Building Communal Practices?

DW’s position between Sean and Uma might have been more open for either student to answer his original question at the start of this interaction than his position (nearer to Sean) from interaction 4. DW’s question and gestures did not indicate that he was talking directly to either Sean or Uma, nor did Sean’s actions suggest he wanted Uma to respond. Instead, Uma heard the question and responded. From there, DW continues to talk to both students (although pointing to Uma’s paper), suggesting the conversation is inclusive to all three individuals – a phenomena that is different from the previous interaction with DW.

Interaction 12: Session 1 (33:51 – 35:00)

Sean: But we do agree that like this block has more time in it than this block—

Uma: Yep, you definitely yeah. So the scale, the number, within it is not the same, yeah. Because this one only goes to 10 and this one goes to 100, so with each scale the amount of numbers within it is increasing.

Sean: Presumably then within these there’s like a compression happening also.

Uma: M hm.

Uma: [17 sec, thinking aloud while working on her paper] So then that’s 1000. That’s 1800. 2000 is going to be right about here [inaudible].

*The interaction ends with the two students working independently on positioning dates on the new timeline. Uma can be heard occasionally thinking aloud as she works. The two work independently for nearly 7 minutes without interacting when DW enters.*

Building Significance?

Sean’s initial talk-turn in this interaction suggests that he felt Uma left out an important piece of information about their new timeline when she responded to DW in the previous interaction. He is clarifying that she understands his statement from Interaction 8. In this way, Sean is positioning his own thinking as mathematically significant. Uma demonstrates that she understands Sean’s thinking in her response to the initial question.

Building Communal Practices?

Uma presents an argument to demonstrate her understanding of Sean’s previous mathematical idea (form Interaction 8) that their eras each contain a different amount of time. Uma gave her argument when pressed for clarification by Sean. In doing so, Uma is making her thinking/understanding open, while Sean has not be held to the same standard (he uses statements, not arguments).

Interaction 13: Session 1 (41:36 – 44:15)

DW: Can I bother you guys to talk? [DW is positioned between Uma and Sean]

Uma: [laughing]

Sean: I think we’re just kinda filling in dates now.

DW: Ok.

Sean: We’ve kinda settled on our—

DW: Ok, so how are you working then? You were trying to figure out how to live/ or how to work within each of your chunks; within each of your eras. How are you working within those then?

Sean: Uuuhh. I guess we could collaborate on that [looking to Uma].

Uma: Yeah.

DW: I mean they appear—your two timelines appear to be the same, so I think you are working—

Uma: Because I think, because the way I was looking at it is I kinda just like worked within—like once we had decided what our like time was, you look at the points within that time. And then I kinda like put them where I thought they would go within that time, knowing that it’s not going to be like accurate and there’s a lot more cl[umping]—like in certain eras you can see them bunching together a little bit more. And because it’s not like, this, the amount of, like because those look equal but they don’t represent the same amount of time.

DW: Ok, and then—

Uma: Because knowing that, that kinda skews the—

DW: So does that work—If I’m, if I’m understanding your idea of skewed here, then this green dot just to the left of 10 to the 9th and the one next to it, those are like one grid space apart, but then you have a similar spacing over here in the interval between 10 to the 8th and 10 to the 9th – so what were you saying about—so this one grid block compared to that one grid block

Uma: Is not [simultaneously with DW]

DW: Is that they skew that you’re talking about?

Uma: Yeah because it’s not going to be—because in between this there’s only 10 and then in between here, this is 100, so like each time the scale—like the amount of numbers in between each is increasing. So even though it looks like on here that they’re equal distance apart they’re not actually equal distance apart in time.

DW: Ok. So the passage of time here is not [simultaneously with Uma]

Uma: Is not the same, yeah even though it looks like it is, it’s not. But that’s the way that our scale kind of makes it seem, which is kinda how it fell.

DW: Ok. Ok [looking back and forth from Uma to Sean]

Sean: I agree with that. I was relying a little bit on, on feel [inaudible]

Uma: M hm, it’s just kinda where it fell.

*The interaction ends when DW changes the focus of the conversation from how the timeline is structured to a specific date on the timeline.*

Building Significance?

Uma positions Sean’s point from Interaction 12 as significant in her response to DW’s question. From there, DW’s persistent clarifying and revoicing position Uma’s argument about the behavior of their new timeline as significant.

Building Communal Practices?

DW was again positioned between Uma and Sean and again did not ask a directed question to either student. Uma responded (again). The last 5 talk-turns suggest that the conversation was open to both students, and that although Sean was not immediately responding to DW, he was paying attention to the conversation and felt represented.

Interaction 14: Session 1 (44:15 – 46:21)

DW: Do you [to Sean] have 500 plotted on yours here?

Sean: So this line continues down here, and 500 is here.

DW: [pointing back and forth between Uma and Sean’s papers] So you both have the 500 the same space, can you tell me how you decided to place it there?

Uma: Because it was kinda the half-way point between 100 and 1000—it just kinda because it’s 500 so it’s half-way between.

DW: Ok, so half-way within—

Uma: Within that chunk.

DW: 500 is half-way

Uma: Yeah, looking at that in isolation of the rest of them [the other chunks] that one was half-way.

DW: Ok, so then are you guys satisfied then with the model that you’ve come up with? You understand it—

Sean: I’m pretty happy with it. I think that um, within each of these blocks we know that—again it’s going to be—there’s more time in this segment than there is in this segment—

Uma: M hm, yeah

Sean: -- you know this whole thing is, is like that, so to put it in the middle is not going to be—

DW: Ok, so I think that’s different [pointing to Sean’s paper] from how I thought Uma was talking a moment ago. I thought she was talking like the two or three blocks here compared to 2 or 3 blocks in the next era as being a different representation of passage of time even though it’s the same—

Uma: Space within there, yeah.

DW: Is that still true within?

Sean: That is still true within. I think that um, and we haven’t really discussed this, but this is kinda my—If I were to graph this, say um I would have, you know 10 and then the next one is going to be 100 or whatever it is, it’s going to be a curve and I think that curve is continuous all the way through and existing within.

DW: So in your mind it does, that curve continues within each of those eras.

Sean: It does.

DW: Does that—I think the two of you need to talk—

Uma: Yeah. A little

DW: -- because you might be thinking a little differently in there [walking away].

*The interaction ends when DW leaves the conversation. The next interaction begins immediately while Sean and Uma discuss the differences in the way they are thinking about the scales within their eras.*

Building Significance?

Similar to Interaction 12, Sean seems to agree with what Uma is saying in her response to DW’s questions, but also feels that she is leaving out aspects that he considers to be important. His thinking about the within-segment representation of time is positioned as mathematically significant in this interaction. DW also positions Uma’s mathematical thinking about the within-segment representation of time as significant by indicating that the two approaches are different and that the pair should talk.

Building Communal Practices?

Same as previous interaction.

DW works to build collaborative environment between the two students with talk-turns at the end of the interaction that suggest Uma and Sean “need to talk” because they “might be thinking a little differently” about the nature of how time is represented within their segments.

Interaction 15: Session 1 (46:21 – 50:57)

Uma: Ok, so you’re saying within---

Sean: Yeah, so we’ve, we’ve chosen these boundaries um, and you can think of those as linear

Uma: Yeah

Sean: Which is kind of how I treated them. Um but, I kinda feel like [8 sec] to like [5 sec] I don’t know how to describe what I’m thinking. I just feel like there’s got to be a relationship, or there could be a relationship between them. Um. [Sean falls silent, looking at his paper and thinking for about 15 sec]  
  
*Uma is listening in on a conversation between DW and two other students.*

Sean: So maybe, what truly—here we have 10 to the 1, right, and 10 to the 2, so maybe what truly lies in the middle here is 10 to the 1.5. Whatever that is.

Uma: Oooooohhh. That, yeah, ok, yeah, and I see that where I was trying to figure out these ones they got a little tricky because like these ones over here they were easier, but these ones when you got like 2 times 10 to the 6th it was hard to determine where that was because it’s different than 10 to the 6th and 10 to the 7th. So it’s hard to find that, where it fits within this. So that [Sean’s idea] does make sense.

Sean: Yep. Yep.

Uma: So then, yeah. So then each one will be—

Sean: So then, err…

Uma: So then is 10 to the 1[point 5], is 10 to the 1.5—what is that?

Sean: I brought my calculator—So that is, is 30. So what we’re saying is that 30 is half-way between 10 and 100 on this scale.

Uma: Oh ok.

Sean: I’ll go ahead and call it 30, but…

Uma: Yeah. So then that would…

Sean: So here is 10 to the 2.5 [pointing on his paper], whatever that is. It’s 316 and we’re seeing how—

Uma: It’s the pattern of, it’s going to go up by a point, so the next one should be— 3 thousand…

Sean: So that’s 10 000 so this is 31 000, right, no that’s—

Uma: I’d be—

Sean: --no that’s 100 000.

Uma: Yeah, I was gonna say I’d be 3000 and then 30 000. 31 thousand

Sean: [getting quieter] This is 3 hundred. And then that’s [inaudible, talking to himself while using his calculator. Uma is leaning over to watch]

Sean: And then you could even go so then really what we need to do is divide this into like 10 segments and then it would be 10 to the 4.1, 10 to the 4.2, 4.3 for each little…

Uma: Yeah

Sean: That would give us—at least then you could pick a point on the line and say, this is that year.

Uma: Yeah, then you, it would, it would give you, it would take it from that oh guessing to a little bit more like it’s gonna fit. Because now you know this point is this and then you can look at what those points and then you can plot them better. That makes sense. Because I didn’t even, that makes sense because I hadn’t even thought about it like that.

Sean: Well I was just kinda like, well I think I was kinda aware of it but didn’t –

Uma: Yeah, you weren’t quite sure how to like—

Sean: Until we just happened on that, and that’s when it kinda made sense. So we could like, we could pick one of these eras 10 to the 7th and 10 to the 8th and we could easily expand that out and make it precise.

Uma: Yeah. Oh ok.

*The interaction ends when DW informs all students that time is running out for the session.*

Building Significance?

Uma regularly revoiced, agreed, and accepted Sean’s statements about the composition of their timeline within segments (endpoints of powers of 10). As with previous interactions, this shows that Uma is positioning Sean’s mathematical thinking as significant. Moreover, in providing the next value in the pattern and supporting Sean to write the appropriate value, Uma is showing that she understands Sean’s mathematical thinking and is willing to accept it as an approach to creating their model.

Building Communal Practices?

Similar to previous interactions.

There are differences in the way Uma and Sean approach their in-progress thinking. Uma seems comfortable inviting Sean into her thoughts as she works to formalize them, while Sean appears hesitant to share thinking that he believes incomplete. The concept of “within-segment” passage of time was brought up in interaction 10, nearly 20 minutes prior to Sean discussing it in this final interaction of session 1.

Interaction 1: Session 2 (1:56 – 4:00)

Sean: So we had kind of agreed that like halfway between that is 10^6.5. Is that what we kind of—

Uma: Yeah, I think that’s, that’s the farthest of where we ended last time.

Sean: Yeah. So then we would have to equate each value to—

Uma: To that, to that interval instead of where we were.

Sean: Which presents us with a new problem here.

Uma: Mhm, yeah.

*Long Pause – Students work independently on their own papers for about 1.5 minutes*

Sean: So [looking up from his paper]

Uma: So we want to break it up into 10, right?

Sean: Yeah, that’s what I’m doing.

*The interaction ends when DW enters to ask a question.*

Building Significance?

In this interaction, Uma agrees with Sean’s argument about needing to change the form of each of the events such that they could be represented as powers of 10. In her agreement, Uma revoices (or finishes) Sean’s sentence. After the two had worked independently for a while, Uma also clarifies their approach (“So we want to break it up into 10, right?”). Together these uses of language, along with Uma implementing the strategy, positions Sean’s thinking as significant. Uma agreeing with Sean that “[we have] a new problem here” also positions Sean’s thinking as significant.

Noting a pattern from Uma’s use of language: Agreeing, Revoicing/Finishing, Clarifying, and Accepting/Implementing with agreement.

Building Communal Practices?

To begin the interaction, Sean recaps a mathematical idea that he authored from the end of the previous session, and then poses a question to Uma. In doing so, Sean invited Uma to join him in recapping where the two had ended the previous session.

Interaction 2: Session 2 (4:00 – 5:14)

DW: Are you guys recreating or just completing? [beginning the conversation from Uma’s right and moving to between the two students]

Sean: Well, I’m recreating. We found an issue. We kind of agreed that between 10^6 and 10^7 that halfway is 10^6.5. But then that involves converting all those numbers to 10 to the power. [DW leans over to see Sean’s paper, Uma continues to work on her own paper]

DW: Oh [standing up and nodding]

Sean: --in order to accurately place them on the line.

DW: Ok. So are you going with that conversion, or are you avoiding that conversion? [Uma leans over to see Sean’s paper]

Sean: [Looking to Uma] I don’t think we’ve really discussed that.

DW: Ok, so you presented that as a problem, so I didn’t know which avenue you were going with. It looks to me like you’ve got it this way [the original way] more readily. So I am interested in hearing your conversation about which one you would choose, but maybe for now in the interest of time—

Sean: time wise— just stick with that [avoiding the conversions]

DW: Yeah just with the one that you have more—and you can weigh the pros and cons of that.

Sean: Sure.

*The interaction ends when DW leaves the conversation.*

Building Significance?

The change from “I’m” to “We” in Sean’s first talk-turn of this interaction suggests a collective sense of ownership about the mathematical idea for converting all dates to powers of 10. This idea was originally authored by Sean; however, Uma agreed with it and positioned it as significant at the end of session 1 and so far in the beginning of session 2.

DW’s suggestion to continue with the most complete model may be positioning that approach as more mathematically significant than the new approach (which would be completely exponential/logarithmic).

Building Communal Practices?

DW poses a question to check in with the pair’s progress, which is answered by Sean. During the conversation, Sean looks to Uma stating “I don’t think we’ve really discussed that” which could be a subtle invitation for Uma to join the interaction.

Interaction 3: Session 2 (5:14 – 8:20)

Sean: So the question I’m kind of coming up with, I guess if we go this route and we convert everything to a power of 10 is, is um “what does it mean when I measure from here to there?”

Uma: M Hm.

Sean: Right, how do we convert that to--?

Uma: Yeah. [pointing on Sean’s paper] Because even still you’re going to have this interval distance is still going to be different, then—

Sean: [nodding agreement] It is

Uma: -- you’re still going to get that problem within each interval compared to the other intervals.

Sean: Because this is [looking at his paper]—I have to refresh my memory on exponents but this is that. That’s not that because this would be 10^16, right?

Uma: Uhhh, I think yes. I think so.

Sean: So is that—What is that? I’m trying to figure out how to go from here to there.

Uma: to there [said simultaneously with Sean]

Sean: Yes. It’s 10^8 times

Uma: Yeah, because you’re going to do—M hm. Because you’re still going to have- because your middle point between 10^8.5 and 10^9.5 this is still going to be a different amount than that [pointing to different intervals on Sean’s paper]. So if we wanted to make it like that, then we couldn’t use the 10 to the exponents. Or, you would just have to know that within each that they’re not equivalent distances.

Sean: I think we need to keep that in mind, and then maybe um maybe part of our timeline would be a key or something. So we could have a formula between date A and B is this many years, which you should be able to do by subtracting, right.

Uma: M hm. Yeah.

Sean: So

Uma: just theoretically by looking at it. Because even though you don’t- like it doesn’t look the same if you focus on 500 in compared to, you know, your 1800 over here even if your distance isn’t really showing you would still know how much time is in between them—

Sean: Yep [nodding]

Uma: -- even if this doesn’t really represent it.

Sean: Like 100 years here [pointing at his paper] is going to look different than 100 years here.

Uma: M hm. Yeah. Exactly.

Uma: [Thinking aloud] So explain how it works. How we created it.

*The interaction ends with each student working independently to answer the prompts on the daily problem sheet (for about 25 seconds). Uma can be heard regularly thinking aloud.*

Building Significance?

This interaction begins with Sean stating a question that he is considering as he thinks about which approach to move forward with. After sharing his question, Uma presents an argument for why Sean’s question is valid to worry about. Uma argues that regardless of whether they convert all values to powers of 10 or stick with their original approach, they will still have to consider different intervals on their model representing different amounts of time. Presenting this argument is a way of validating Sean’s thinking, and thus positions Sean’s thinking as significant.

Sean agrees with Uma’s argument and offers a method for “keeping that in mind” with a key/legend for their model. Then, he revoices Uma’s argument by presenting a specific example. Agreeing with and revoicing Uma’s argument positions her mathematical thinking as significant in this interaction.

Building Communal Practices?

Sean originally states the question that he was posing to himself while critiquing the new approach of converting all values to powers of 10. Uma took this opportunity to present an argument about how this new approach would still present similar issues as their previous approach regarding the length of intervals on their timeline and the amount of time those represent. Sean acknowledges her argument by agreeing with it, but when Uma continues to present further evidence to her argument, Sean seems to be focusing on converting values over hearing Uma’s evidence (at first). Interestingly, Sean appears to be closed off from Uma’s argument while also being inviting about how to convert values from scientific notation to powers of 10. Then, Sean revoices Uma’s argument through an example.

Uma’s presentation of her argument and continuing to interact with Sean’s questions about converting are both open approaches to inviting collaboration. ON the one hand, Uma is inviting Sean to interact with her thinking about why converting to powers of 10 would still present similar issues that scientific notation has presented. On the other hand, Uma is also taking Sean’s invitations to interact with his thinking about carrying out calculations. Thus, Uma is open on two fronts throughout this interaction, while Sean is only focused on calculations until he revoices Uma’s argument, which effectively works to end the interaction.

Interaction 4: Session 2 (8:44 – 9:54)

Sean: So initially we kind of arbitrarily divided the dates up into like four sections or something—

Uma: M hm [affirmative]

Sean: -- it was just kind of

Uma: Yep, it was just kind of a guess.

Sean: Yep. Yep. Then we realized- and I’m, I’m saying that this is what we did but if it rings not true then tell me –

Uma: M hm

Sean: --but then we kind of realized that we were still having that clumping problem.

Uma: Yeah, so then we tried to come up with different- we tried to use different intervals. And then we decided to, instead of—because before we were breaking it up like, the in—we were breaking it up by like each like a number of how many, so we went like five to each group. Yeah, and then you switched it up to grouping it by the exponent – 10 to the whatever – so you could get a better representation.

Sean: [nods silently while looking at his paper]

*The interaction ends with the pair continuing to work on their own papers. Sean appears to have been working throughout the interaction, never looking at Uma while she was talking. They work independently for more than 3.5 minutes.*

Building Significance?

In this interaction Uma listens to and agrees with Sean’s initial recap of their initial approach and the clumping problem they encountered. Then, Uma repeats Sean’s recap and states what “you” did next and why. By repeating Sean’s original statements and then building from those, Uma positioned Sean’s thinking about their approach to creating a timeline as significant. Also note Uma’s use of “you” instead of “we” when describing their approach to Sean. This also serves to position Sean’s thinking as significant.

Building Communal Practices?

Sean invites Uma to disagree with him if his recap of their evolving strategy did not “ring true” for her. Uma takes Sean’s invitation to collaborate, but does so by agreeing with Sean’s recap, repeating it, and stating what “you” did next and why. Uma’s use of “you” instead of “we” could be an attempt at inviting Sean to continue to collaborate based on Sean’s thinking. Though, Sean does not take up the invitation. Sean nods agreement silently and continues working on his own paper for well over 3.5 minutes.

Interaction 5: Session 2 (13:30 – 15:40)

Uma: [sighs very loudly]

Sean: [looks towards Uma and then back to his paper]

Uma: [inaudible] the time the dinosaurs existed. So we need to find the distance between Jurassic and Cretaceous? [turning to position her papers between her and Sean]

Sean: Yep. [looking towards Uma’s papers and then back to his own]

Uma: So this amount of time?

Sean: Yep. [looking at his own papers]

Uma: And then the rest of—[reading from task sheet] compared to the existence of humans. What are we saying, when are we saying existence of humans started? Oh, right here at the Pleistocene [mispronounced].

Sean: Pleistocene [pronounced correctly]

Uma: Pleistocene [pronounced correctly] it says “Modern Human Beings Developed.” So are you saying it starts there?

Sean: Yep. [continuing to work on his own paper]

Uma: OK. So we need all that [Sean looks at Uma’s paper]. [Getting louder] So we’re finding that distance of this compared to that; the amount of time in between each.

Sean: Yep. And so on mine, I’ve got [writing on his paper], I’ve got 2.08 and 1.44 times 10 to the 8th [pausing while Uma writes on her paper].

Uma: [Quietly] Right here, 2 point 08 [louder] OK, so I have this amount of space for those ones [pointing on her paper]. And Pleistocene is 2 to the 6th and that is from here all the way to ‘Now’. So—

Sean: So on our, on our, it’s very very tiny—

Uma: M hm, it makes it look like this is a longer period of time. I don’t know if that’s true, but that’s what it appears like.

*DW walks over silently to stand between the two students [15:38]*

Sean: We know that down here more time is compressed into the same amount, so...

Uma: M hm, yeah.

*DW enters the conversation, which ends the previous interaction and starts a new one.*

Building Significance?

Neither student establishes the other’s thinking as significant in this interaction. Interestingly, Sean does not share his thinking (or new thinking) in this interaction until DW walks behind the two, so Uma does not have any thinking to position as significant. On the other hand, Uma states her thinking about the problem a couple of times, and Sean responds by stating the values he was using to consider the *existence of dinosaurs*.

Building Communal Practices?

Uma issues multiple invitations for Sean to begin collaborating through thinking/reading aloud and asking questions at the start of this interaction. All of which were only acknowledged by Sean. Sean did not begin to more fully participate with Uma until he was ready to state the dates he was using for *existence of dinosaurs*. Until that point, his only involvement with Uma was to correct her pronunciation of Pleistocene.

Interaction 6: Session 2 (15:40 – 16:38)

DW: Can you say that again? Sorry.

Sean: So what we’re discussing is, um, between the Cretaceous and Jurassic on our scale is small, whereas modern humans is huge. But we know that down towards the left end of our scale, the later um that there’s more time compressed into the same physical area. So—

DW: OK [nodding]

Sean: We expect that- well we know that it may not be as small as it looks.

DW: Oh OK [pointing to Sean’s paper]. So the distance is not—

Sean: -is not-

DW: -- indicative of time.

Sean: No. no.

DW: And you were saying that as you move to the left is even less indicative of time as you move [?]

Sean: It’s getting more and more compressed. More years per square.

DW: [nodding] More years per grid block.

Sean: Yep.

Uma: M hm

DW: OK [walking away]

*DW walks away, which ends the interaction.*

Building Significance?

From the previous interaction, Uma stated that the difference in scale on their model might have implications about which period of time is longer, regardless of the size of the interval on their timeline. Then in this interaction, Sean presents that understanding of their model as data to support a claim about why “we” suspect that the interval reflecting the amount of time for the existence of the dinosaurs “may not be as small as it looks.” Sean uses “we” in his re-presentation of their thinking about the prompt, which could be working to build Uma’s thinking from the previous interaction as mathematically significant – although this was shared thinking. Then, DW clarifies and revoices their argument which works to position their collective thinking as mathematically significant.

Building Communal Practices?

DW invited Sean to re-present the pair’s thinking about the prompt comparing the amount of time between the existence of dinosaurs and modern humans. DW’s invitation was issued through asking Sean to restate something he had just said to Uma. The invitation in this interaction is not all that interesting. Instead, it is the lack of an invitation to Uma for her thoughts that is interesting. DW speaks only with Sean and points to Sean’s paper during the interaction. Only near the end of the interaction does DW look towards Uma to acknowledge her presence in the space.

Interaction 7: Session 2 (16:38 – 18:36)

Sean: So we’re going to do the math to see what it actually looks like, right?

Uma: Yeah.

*DW interrupts everybody with some instructions about where to go prior to a whole group conversation. During this time, Uma and Sean are working on their own papers to “do the math.” The interruption takes about 40 seconds. The pair continues to work silently for an additional 40 seconds*

Sean: If that’s easier, then I’ll let you use that [handing Uma his calculator]

Uma: Oh yes, probably. Sometimes the phones just don’t have all the same—

Sean: Well, they’re a phone, but they do pretty good, right?

Uma: M hm.

*The interaction ends with the pair working independently for 1 minute 50 seconds, and then DW calls the whole group to attention to launch a discussion about the existing solutions so far. The whole group conversation lasts roughly 10 minutes.*

Building Significance?

Sean states the direction he wants the pair to take to continue answering the prompt about which interval represents a longer amount of time to begin this interaction. Uma agrees acknowledges his statement, and implements his suggestion on her own paper. Uma positions Sean’s thinking as mathematically significant by Agreeing with and Implementing Sean’s suggested strategy.

Sean does not position Uma’s thinking as mathematically significant in this interaction.

Noteworthy: While working to “do the math” Uma is using her cell phone to perform calculations.

Building Communal Practices?

Sean issues two “invitations” for Uma to work independently in this interaction. The first invitation takes place with his stated recommendation (phrased as a pseudo-question) that they should “do the math” to compare the amount of elapsed time between to two events. The second “invitation” occurs when Sean hands Uma his calculator to “allow” her to finish carrying out the calculation independently. Moments later, Sean can be heard stating the result of his calculation, which suggests he had already completed the same calculation that Uma was working through. “Allowing” Uma to use his calculator, together with his choice not to share his results of the calculation, imply an effort against collaborating on Sean’s part.

Uma’s response to Sean’s second “invitation” appears to be an effort for her to continue discussions with Sean. She seems to accept that he is unwilling to collaborate about the problem, so perhaps he would be willing to chat about the calculator functionality of cell phones.

Interaction 8: Session 2 (30:03 – 34:12)

Sean: So you see that 33.8 here? That equals 1.5 times 10 to the 10th.

Uma: Yeah, times 10 to the 10th.

Sean: So he’s kind of figured out, ‘What can I raise to the 2 to get that?”.

Uma: M hm. Yea. And you can still see this, that similar to ours is that all of them- well most of them are contained within this top [short pause] section.

Sean: [very quietly, while working on his calculator] I’m kind of curious. So that’s 8, and then [inaudible], so it seems like it’s linear. [continuing to work on his own paper]

Uma: M hm. [leaning over to see Sean’s paper]

Sean: But then [12 seconds. Sean is working on his own paper] – So I’m curious how many—These are- that’s linear- 8 units away, 8 units away

Uma: M hm.

Sean: But how many years are between those?

Uma: Yeah, what’s the distance? Is it, does it come across the same problem that we did with the distance?

Sean: [Continuing to work on his calculator] It’s going to.

Uma: M hm. Yeah.

Sean: So, 2 to the 30th minus 2 to the 22nd [saying the calculation he’s doing on his calculator] is, whatever that is. I need to change my calculator so it’s—[15 seconds. Using calculator] – way smaller.

Uma: Yeah so they still- that still deals with the same problem of the spacing.

Sean: [nods silently]

Uma: -- where the spaces look equal but they’re not. Because in order to make it equal, I think you would need to make it a lot bigger of a graph.

Sean: Well and you’d have to, you’d have to um get rid of the whole exponent thing [looking at his own paper].

Uma: M hm [leaning towards Sean].

Sean: Here’s probably a good thing, I don’t know if it was on the list or not but last year is here [pointing on the hypothetical timeline]. That’s actually a pretty big fraction actually down there [looking at his own paper].

Uma: Yeah.

Sean: Because I suppose this is 2 to the 1, so that is two years.

Uma: M hm [4 sec]. But then yeah, but then that next one’s going to be 4 years [8 sec]

Sean: [quietly] 4 years, now here’s, so there’s…

Uma: 4 years from that to 0—

Sean: Right

Uma: But still only—

Sean: So there’s two years in that

Uma: So those are equival[ent]—

Sean: Right, but this is 8, so there’s four years in that. [nodding. Looking at his own paper] 16 in there.

Uma: M hm. And then you’re slowly getting up to more and more.

Sean: Oh, that’s going to be 8 in there I guess, but yeah, it’s making- we can see what’s [stopped to do a calculation]

Uma: M hm. [after 12 seconds, changes from looking at Sean’s paper to looking at her own]

*The interaction ends with about 15 seconds of silence, where Sean is still looking at his own paper and occasionally writing on it. When the pair begin discussing again, they are talking about a different prompt on the task sheet, which delineates the two interactions.*

*Interesting to note, Sean did not ever look at or towards Uma during the interaction. They talked for over 4 minutes without Sean making eye contact.*

Building Significance?

Sean begins this interaction by drawing Uma’s attention to his realization about the construction of the hypothetical student’s model, and then makes a series of statements – some of which appear to be to himself. Then, Sean asks, “But how many years are between those?” which opens an opportunity for Uma to position Sean’s question/mathematical thinking as significant. Uma’s response to Sean’s question serves three purposes: (1) it demonstrates that she agrees that Sean’s question is mathematically interesting, (2) she revoices Sean’s question, and (3) she makes connections between Sean’s observations of the mathematical properties of the hypothetical model and their own. These three uses of language position Sean’s mathematical thinking as significant. As a response, Sean quickly acknowledges Uma and then continues to complete calculations he finds interesting.

Sean then presents an argument based on his calculations that intervals of similar length on the hypothetical model are “way smaller.” Uma extends Sean’s argument by using it as data to support her claim that the hypothetical model exhibits similar properties to their own timeline. Sean silently acknowledges Uma’s argument. Extending Sean’s argument and building on it to further the pair’s understanding is positioning Sean’s thinking as significant.

Following Uma’s argument, she makes a statement about how to avoid the “spacing issue.” Sean responds with a statement and then continues stating his own observations about the hypothetical model, leading to his presentation of an argument based on more calculations. Once again, Uma agrees with Sean’s argument and then extends his ideas to further unpack the properties of the hypothetical model. Similar to before, Uma’s extension of Sean’s argument positions Sean’s thinking as mathematically significant.

The final segment of the interaction, Uma makes a statement based on one made by Sean about what portion of the hypothetical model is accounting for 4 years. Sean uses Uma’s contribution – that the interval from 4 to 0 represents 4 years (as opposed to between 2^2 and 2^1) – as data for his claim that there are only 2 years in the 1-unit space between 2^2 and 2^1. Building from Uma’s contribution positions her thinking as mathematically significant.

The rest of the interaction concludes with Uma making additional arguments [attempting] to build from the previous contributions that are met by statements from Sean. Moreover, Sean’s statements do little more than acknowledge Uma’s contributions.

In sum, Uma positions Sean’s thinking as significant on three separate portions of this interaction using two different discourse patterns. First, she agrees with, revoices, and makes connections between Sean’s thinking earlier in this interaction with the pair’s previous work. Then on two occasions, Uma includes Sean’s previous statements as a component of an argument that extends his ideas to further make sense of the new, hypothetical model and associated prompts. Sean positions Uma’s thinking as significant once, using a similar pattern to Uma by extending her contributions as a part of making an argument.

Building Communal Practices?

Sean’s use of language and continuous focus on his own paper and work was closed to collaborations throughout this interaction, with the exception of when he used one of Uma’s contributions to build an argument. Though, when Uma attempted to continue the argument, Sean changed the focus by making statements about his own observations or short acknowledgements. In fact, throughout the interaction Sean seemed to be narrating his own thoughts instead of attempting to collaborate with Uma, evidenced by how Sean never made eye contact with Uma, frequently spoke quietly as if thinking aloud, and continuously reverted back to determining values on the hypothetical timeline (even after it was clear both he and Uma understood some of the features).

Alternatively, Uma’s use of language and actions routinely suggested that she wished to collaborate with Sean about his observations/thinking. To begin, Uma positioned herself to look at Sean’s paper. Then, in many contributions Uma either agreed with, revoiced, or built from one of Sean’s contributions.

Interaction 9: Session 2 (34:13 – 36:50)

Uma: Um, let’s see, so then if we were to look at modern humans, from there all the way to here… [6 seconds] Cretaceous to um…

Sean: This isn’t labelled very well, right.

Uma: Yeah. They have, they have different labels. Well they just used these. So you want dinosaurs the largest to the—

Sean: so dinosaurs with— [looking on his own paper]

Uma: --the other dinosaurs. So this space. [looking on her own paper]. So we want this [quietly]  
  
[15 seconds. Uma sits back in her chair to look on Sean’s paper]

Sean: 2 to the 27.63 minus 2 to the 27.1 [43 seconds] I don’t know if there’s any real, like when we get right down to it between- [tilting paper so Uma could see] So ours we could have described as 10 to the t, right? Because we were using…

Uma: M hm. Yeah. [nodding]

Sean: Um so is there really any advantage? I don’t know if there is.

*DW Enters, which creates a new interaction. Of note, DW was originally physically placed nearer to Uma, then between the two students, and finally nearer to Sean before asking a question.*

Building Significance?

Neither student positions the other’s thinking as mathematically significant in this interaction. Instead, they are working to understand the prompt and identify key information from the hypothetical model.

Building Communal Practices?

Uma opens this interaction by inviting Sean to collaborate on the next prompt from the session 2 task sheet. Sean and Uma continue to think aloud for the next couple exchanges, not really clear that either student is specifically talking to the other. Then, Sean invites Uma to see his thinking about the similarities between the hypothetical model and their model through statement and repositioning his paper for her to see more clearly. Uma acknowledges Sean’s statement, but then the conversation is cut short by an interruption from DW.

Interaction 10: Session 2 (36:50 – 39:10)

DW: Does this one have the within exponent sit[uation]?

Sean: The within?

DW: situation that you guys had? That was something you guys had really spent a lot of time talking about. Because you started with eras and then you moved to this exponent on 10. Right, and now you’re looking at within- like over there you were looking at within each segment wondering how to position things.

Sean: OK [nodding]

DW: Does this student have a similar dilemma?

Uma: No [3 seconds]

DW: I mean I’m not asking to infer some answer. I’m just trying to assimilate your experience with building your model to their—

Uma: Yea—

Sean: I, I –

DW: -- to try to think through what this student might have encountered, if they thought about that stuff.

Sean: I think there is in a sense that we’ve got—for us that [pointing at his paper] was 10 to the 5th and 10 to the 6th, or whatever it was, and we were struggling with like where does birds fit within there. So he’s got 2 to the 25th and 2 to the 26th, and he still had to figure out um where does that belong in there. But he did it using— those numbers

DW: using those numbers [simultaneously with Sean]. Right.

Uma: Yea.

Sean: and we never made it to those numbers to converting—

Uma: M hm. We just stuck with—

Sean: 2 time 10 to the whatever it was. We never solved for that.

DW: Oh OK.

Sean: Um. I think is what this student has done. Has solved for t and used it to put things kind of on a linear—[pauses abruptly for 8 seconds] So they’re not, he’s not putting them on a line of 10 to the t. He’s putting them on a line of just that exponent I think.

Uma: M hm.

DW: OK, I see you think this is—

Sean: that’s the exponent.

DW: -- just the exponent associated with a base of 2?

Sean: Yeah, I think so.

Uma: Yeah, because he did that with, because he went from 0 all the way, and like as you can see some of these did have like—

DW: Any events on there

Uma: M hm. Yeah.

DW: OK.

*DW leaves the conversation, ending the interaction. Sean and Uma then work independently for about 3.5 minutes. The pair are working through the hypothetical model’s version of comparing the amount of time for the existence of dinosaurs to that of modern humans. They do not talk while working. They do not discuss their results when done. Sean appears focused on his own papers and completing the problem. Uma is seen looking around at other groups and looking onto Sean’s paper while she works.*

Building Significance?

DW positions thinking as mathematically significant in two different instances during this interaction. First, DW re-presents the pair’s original thinking about position dates on their timeline model in comparison with how the hypothetical student completed the new model. Doing so positioned the pair’s collective work as significant, and worthy of consideration/comparison. Then, DW clarifies Sean’s statements about how he believes the hypothetical student created “his” model, which Sean described as similar to their approach. Repeatedly clarifying Sean’s thinking could have worked to position his thoughts about how the hypothetical student’s model was constructed as mathematically significant and worth discussing.

Building Communal Practices?

DW initiated the interaction with a question inviting the pair to think about and discuss the similarities and differences between their approach to creating a timeline model and the hypothetical student’s approach. DW was positioned nearer to Sean, and although Sean was the main contributor of his thinking, Uma did acknowledge statements and responded throughout the interaction.

Interaction 11: Session 2 (42:30 – 48:22)

Uma: OK so, I was starting to look at this one. This the third question.

Sean: [reads the prompt quietly] So what events are you picking?

Uma: Um, I had thought about doing the Mayan Civilization, and [inaudible]

Sean: OK

Uma: Yeah, so that this one and then, I was thinking the this one over here – the Rodents, Cats, and Dogs.

Sean: OK, and then what are we doing with those [looking at the prompt]?

Uma: [reading the prompt aloud] Compare the amount of elapsed time… OK, so then comparing the amount of time between these ones compared to the amount of time between these ones.

Sean: So the time in between those two.[?]

Uma: M hm

Sean: And the time since the development of Euclidean Geometry? [looking at the task sheet. 16 sec]. So between those two is 2 to the 25.14 minus 2 to the 9 point [inaudible] [Sean gets quieter and is working on his own paper. Uma is leaning towards him]

Uma: [repeating Sean] 2 times [sic] 25.14 minus 2 times—

Sean: 2 [loudly] TO THE 9.966.

Uma: [quietly] point 9 6 6.

Sean: So that’s what’s in between those two.

Uma: M hm. And then the other one’s going to be 2 times—2 to the power of 9.966 minus 2 point [looking at task sheet, rereading the prompt] Compare the amount of time…

Sean: I think it’s just since—

Uma: Oh, so the time elapsed since the development… [rereading the prompt] OK so that would just be 2 to the power of---

Sean: 10.97.

Uma: 2 to the 10.97. And comparing those two distances?

Sean: M hm. [16 seconds. Working independently] Yikes. It looks like 3, 36.97 times 10 to the 3rd, or 10 to the 6th.

Uma: [Looking onto Sean’s calculator] 3, 36.97 times 10 to—

Sean: hah, my writing is…

Uma: --- to the—

Sean: 6th.

Uma: 6th, OK. That’s this distance. And then [looking to Sean as he uses his calculator]

Sean: 2 to the 10.97 is 2006 roughly.

Uma: So then you’re comparing [pauses to take the ESM survey]

Sean: And then this is 11 along there, and then this is 25 mmm, is that minus 10[?] so 15 [working on his own paper]

Uma: 15 and 11?   
  
[29 seconds. Uma is looking at the clock and back to Sean. Sean is working on his own paper]   
  
Is that what the question is asking? Is that what…

Sean: The next question is what he’s [DW] talking about. It’s asking about um… [continues working on his own paper]

*The interaction ends with Sean continuing to work on his own paper and Uma reading the next prompt quietly but out loud. Sean was looking at and writing on his own paper for the last 2 minutes of the interaction. Uma was looking on Sean’s paper and her gaze would regularly go to the clock and other groups seated at the table.*

Building Significance?

Sean does not position Uma’s mathematical thinking as significant in this interaction. He implicitly agrees to use the events Uma suggests to complete the problem, but this agreement is not indicative of Uma’s mathematical thinking.

On the other hand, after suggesting the two events, Uma listens to Sean’s approach for finding the amount of elapsed time between them, follows along with his calculation (repeating him often), and then implements his strategy to find the amount of elapsed time since the Development of Euclidean Geometry. Attempting to use Sean’s strategy positions his mathematical thinking as significant.

Building Communal Practices?

After sitting silently and working independently for roughly 3.5 minutes since the previous interaction, Uma invites Sean to collaborate on the next prompt to start this interaction. Sean accepts the invitation, reads the prompt, and then asks Uma what information she was planning to use to start. After hearing the two events Uma picked, Sean’s contributions to the rest of the interaction are either in the form of: (1) stating his calculations out loud, (2) correcting Uma’s imprecise mathematical language, or (3) acknowledging Uma’s contributions without participating in a discussion. In fact, much of Sean’s statements are said very quietly while he is looking at his own paper, where he frequently takes long pauses to complete calculations silently. Except for his first question, all of Sean’s contributions to this interaction are closed and do not promote further discussion.

Uma launches the interaction with an invitation to move forward on the prompts from the task sheet. Then, Uma’s contributions are in the form of: (1) responding to Sean’s questions with statements, (2) repeating Sean to follow along with his calculations, or (3) implementing Sean’s strategy. Uma’s contributions to this interaction are open, and attempting to promote further discussion.

Interaction 12: Session 2 (48:26 – 51:50)

Uma: [quietly reading the fourth prompt] … [getting louder] Which even took place longer ago and by how much? So did that just mean that the one point is going to be over here and the other point is going to be over here [pointing on her paper]? Is that what that—

Sean: [looking back and forth between Uma’s paper and his own] Yep. Yep.

Uma: [quietly] So, B… and this is A. It says, which one of these took place longer ago[?]. I mean it would just be B right?

Sean: It would be, B, yep yeah.

Uma: And that’s just because it’s farther back on the timeline.

Sean: [nodding, using his calculator] Yep. And then it’s asking by how much [writing on his paper].

Uma: So then you’d have—you would just subtract 18—[gets quieter and begins writing on her own paper] 18.03 [inaudible] [looks towards Sean’s paper]

Sean: [11 seconds] So that would give us the, the… [using his calculator]

Uma: M hm.

Sean: I don’t know if there’s a simpler way of doing that… [4 seconds] So about 199,000 years. Does that seem right?

Uma: I think so. Does it- so I’m just trying to remember like with exponents, can you, if it had the same like number, you can- can you take- so, like if we were to subtract because what I had done because if you had subtracted the one and then the other one, and if you were to put 2 to the power of that [inflection intonates a question]

Sean: So by inspection that’s going to be—

Uma: be a little bit bigger, right?

Sean: --- be very very small. Because that’s going to be almost 4. Right, because 2 squared. But I think you’re on to something. There’s probably a relationship there. Um [looking at his paper… 40 sec]

*This interaction ends with Sean abruptly stopping his final contribution and looking at his paper while Uma sits silently for about 40 seconds. After which, Uma begins writing on her own paper, and can be heard vocalizing the subtraction approach to determining how much longer ago event B was from event A.*

*Then, the session concludes with a whole group summarization of progress on the prompts.*

Building Significance?

At the end of this interaction Uma can be heard implementing Sean’s subtraction approach to determining the amount of time between events A and B from the prompt sheet. This takes place after she presents an argument for using a different approach, a multiplicative approach that relies on properties of exponents. Implementing Sean’s strategy and foregoing her own recommendation positions Sean’s thinking as significant. That 40 seconds go by with Uma looking on Sean’s paper (without being acknowledged by Sean) before Uma implements Sean’s strategy could be a sign of reluctance.

Sean does not position Uma’s thinking as significant in this interaction. In fact, there are two instances when Uma presents an argument, where Sean’s responses are closed. The first instance, takes place when Uma argues for why event B took place longer ago than event A – Sean acknowledges agreement with one word. The second instance takes place after Uma shares her argument about using a multiplicative approach to determining how much longer ago event B was than event A. Sean responds to Uma’s argument with an argument comparing his solution [199,000 years between the events] to hers [event B was 4 times longer ago than event A]. Sean then acknowledges Uma’s thinking with “But I think you’re onto something. There’s probably a relationship there” and discontinues the interaction. Sean’s limited willingness to further consider Uma’s approach, while simultaneously comparing her solution to his own, suggests that he is positioning his own mathematical thinking as more significant than Uma’s.

Building Communal Practices?

As with the previous interaction, Uma initiates this interaction with an invitation to collaborate on the final prompt. During this interaction, Uma’s contributions are in the form of sharing her thinking through arguments, which frequently intone questioning. Uma’s contributions are open.

Sean’s contributions demonstrate a similar pattern to the previous interaction. He either acknowledges Uma’s contributions without further discussion or argues against her mathematical thinking. Throughout, Sean can be seen working on his own paper and using his calculator to carry out computations. Sean’s contributions are closed. Most notably, Sean demonstrates no interest in further discussing Uma’s idea of using a multiplicative approach to solving this problem (even after searching for a “simpler way”) through his final contribution of the interaction. Instead of continuing to discuss Uma’s method, he brushes her argument aside and continues working on his own paper. Sean’s contributions and actions do not suggest an interest in further discussion with Uma.

Interaction 1: Session 3 (6:15 – 7:20)

Sean: [Looking at Uma] If you multiply 2 times 2 blocks, you get two years, or whatever it is, right?

Uma: [Looking at Sean] Mmmhmm.

Sean: It’s a linear graph that would be a straight line.

Uma: Mmmhmm, yeah.

Sean: It would be a straight line.

Uma: Yeah, you’re getting the same amount of space compared to these ones that weren’t. They were more exponential because the spaces between all represented different amounts.

Sean: There’s more time the further we go down, right?

Uma: Mmmhmm.

[Pair silently looks at the paper between them]

Sean: [inaudible]… we think of time as being linear, so it’s a natural—

Uma: Yeah, a natural reaction.

*Interaction ends with DW addressing the group.*

Building Significance?

To begin this interaction, Sean shares what his thoughts are on the question DW posed to the entire group. Uma responds to Sean’s question (request for confirmation) by agreeing with his ideas and restating them in her own terms, positioning his mathematical thinking as significant. Sean does not interact with Uma’s interpretation of his thinking, but rather poses another question to her.

Building Communal Practices?

During Sean’s statement of his thinking, he ends his statement with a “right?” implying that he is interested in Uma discussing his reasoning. However, based on the way in which Sean had implemented his strategy previously, he was not expecting for Uma’s response to push against his ideas. Rather, his question was more of a request for Uma to reiterate to him that his ideas were correct. Ma then agrees with Sean’s statement and revoices his ideas in her own terms. Sean then asks Uma another question, in a similar way as before, searching for agreement from ma. At the close of the interaction, Sean states why he thinks it would be “natural” for people to think of time as linear, but his statement is closed and does not invite Uma to discuss his ideas.

Interaction 2: Session 3 (12:30 – 14:15)

Uma: So he wants… um, how do you…? Um, so like the concerns for too much paper? Is that what the concerns that we were—like the space?

Sean: yeah, so either the two concerns were either the clustering of having to extrapolate that line out so far. And, you know, our model and the student’s model are really similar, right?

Uma: Mmmhmm.

Sean: [inaudible]

[Sean looks at his cellphone while Uma is talking. Uma looks at Sean and realizes he is not paying attention to what she is saying.]

Uma: It’s just the different interval kind of, yeah. I mean it makes it so that you can get an idea of the whole thing on this one sheet of paper, so it doesn’t feel like too much.

Sean: I think, um…

Uma: I think if it were—if it was linear and was on the whole thing, it might look a little bit more overwhelming with how long it would have to be.

Sean: I think that it’s kind of important this is that in order for us to use our model—the thing that drives you to using our model is that there are so many dates that are recent, right? That there is already kind of this clustering. So, that allows like at the beginning of our scale, it allows us to spread some things out and then towards the end they are kind of compressed.

Uma: Yeah, it kind of puts them back.

*Interaction ends with both looking at cellphones.*

Building Significance?

The interaction begins with Uma asking Sean if her summary of their concern is correct, which Sean agrees to. He then states what he believed the two concerns were, not connecting these concerns with the idea Uma voice, (possibly) inadvertently pushing aside her idea. Uma then agrees with Sean’s summary of the concerns they had, building his ideas as significant. While Uma is describing the similarities between their model and the student’s model, Sean is staring at his cellphone showing Uma that he did not feel that her ideas were significant. Uma then states why she believes a linear model would be overwhelming, to which Sean does not reply. Instead, Sean states why he believes their model is superior to the student’s model.

Building Communal Practices?

The interaction begins with Uma asking Sean a clarifying question as to what the concerns they had when considering their timeline. Uma summarizes what she believes a concern was but asks Sean if this was the concern they had. In doing so, Uma was inviting Sean to interact with her recollection of their previous discussions, and opened the conversation for Sean to become involved.

Interaction 3: Session 3 (20:30 – 26:15)

Sean: I guess we just do this one at a time, huh?

Uma: Yeah, I think so [reads prompt out loud].

Sean: To me, I’m kind of just imagining taking steps…

Uma: Mmmhmm

Sean: …and each step I take is way longer than the last step.

Uma: Yeah, that’s kind of the immediate response is just that it is multiplying, so you are getting exponentially bigger with each interval you go to [reads the next prompt].

Sean: [looking at paper] I think that it is.

Uma: [Sean looks at Uma’s paper while she is gesturing] Yeah, as long as they’re the same—as long as you are looking at the same length, then you’ll always be getting that same like exponential increase.

[6 second pause]

Sean: So, [reads next prompt] I have to draw a little bit to try and…

Uma: I was going to say, “What is that?”…

Sean: So, if this present here, here’s a segment of time, and there’s a segment of time overlapping…

Uma: Yeah.

Sean: I think that’s what it’s talking about.

DW: Yeah, and I gave you an example for you to use.

Uma: Ah, for example…

Sean: Oh, ok.

Uma: [reads the example in the prompt, and talks to herself about numbers]

[50 seconds of silence]

Sean: [re-reads prompt out loud]

[10 seconds of silence before Uma talks to herself]

Uma: [stating her calculations audibly]

[50 seconds]

Sean: What are your thoughts? [looks at Uma, but then down at his paper the entire time she’s talking]

Uma: I’m trying to figure out what it’s—so, it wants you to compare like how many times longer is this from the other segment? I’m trying to figure out what that…

[20 seconds]

Uma: I’m trying to figure out—because within like the amount in between here is different from the amount there. [Sean turns to look at Uma’s paper briefly] So, I’m curious how you relate them if they are in different—they’re overlapping. I guess that’s where I am. I’m at a standstill.

Sean: Yeah, I’m kind of uh… yeah, I’m not really sure, I…

Building Significance?

Throughout this interaction, Uma agrees or accepts the statements Sean makes, signifying his mathematical thinking as moving their group forward. Even when Uma is extending Sean’s ideas, adding her own interpretations, he does not acknowledge her thinking as significant. Rather, often when Uma is stating her mathematical thinking Sean is looking at his cellphone with no outward indication that he is listening to her.

Building Communal Practices?

Sean opens this interaction with a question about how they should address the prompts, which the answer seems fairly intuitive. Sean then continually makes statements about his thinking which are closed and do not invite Uma to reason with him. A notable exception comes at the end of this interaction, when both Uma and Sean are struggling to navigate a solution to one of the prompts. Sean provides Uma with a genuine invitation to discuss her mathematical thinking with him, which is quite different from the previous invitations that are commonly followed by his own stated solutions.

Interaction 4: Session 3 (26:15 – 27:00)

Sean: [to DW] I’m honestly having trouble kind of conceptualizing that problem.

DW: Like my question?

Sean: Yeah.

DW: I like your picture. This definitely captures what I’m asking. So, what I was asking was if these two segments are the same length, but they overlap, is this one still 10 times more time?

Sean: Okay.

DW: Does that make sense?

Sean: Yea, it does.

Uma: Oh, okay.

Sean: It does.

DW: So, you were saying in part b that you think the relationship is preserved for successive segments of the same length. I’m asking, what if they’re not really…

Uma: Successive.

DW: …successive. What if they’re overlapping a little bit? Is it still 10 times the amount of time?

*Interaction ends when DW leaves.*

Building significance?

During this interaction, DW compliments Sean’s picture, positioning his thinking as significant.

Building Communal Practices?

This interaction begins with Sean telling DW that he doesn’t understand “his” question. This question comes after Uma shares her thoughts with Sean and says she’s at a “standstill.” Sean does not interact with Uma about his confusion, instead he turns to DW. During the interaction, DW poses questions to the pair, looking at both Uma and Sean. During one of DW’s questions, Uma interrupts DW anticipating the word he will use next. This action is potentially Uma’s way to become included in the conversations.

Interaction 5: Session 3 (27:05 – 31:00)

Uma: I think so? I’m a little…

Sean: Well, one thing we know is that the area they overlap is the same time.

Uma: Mmmhmm

Sean: So, then the question is what is this time compared to… what is the un-overlapped time on the lower end compared to the overlapped time on the upper end?

Uma: Well, he said that each segment is equal, right? So you’re saying that this part [pointing at Sean’s paper] is the same amount of space as this part, right?

Sean: I think from here there is about the same as there.

Uma: Yeah, okay, so now the question is, When you take the overlapped time, does this still preserve the same amount?

Sean: As that 10 to 1? That is the question. I don’t think that it is.

Uma: Mmmhmm. Do you have any thoughts leaning you more towards why it’s not?

[15 seconds]

Sean: If we graphed them longer, we would end up with something like that.

Uma: Mmmhmm.

Sean: Um, we’re saying then that is equal with that, or this is 10 times that, but I think this…

Uma: That overlap takes away from it being able to be 10 times that?

Sean: Yeah, I just don’t know how to…

Uma: Yeah, how to explain…

Sean: I guess here’s what I would say. We know that if this were true, then shouldn’t I be able to slide it a little bit further and a little bit further, and then I would have them on top of each other, and they are clearly not 10 to 1 then.

Uma: Mmmhmm.

Sean: You know what I mean? If I keep overlapping more and more, then they become the same time period. So, I don’t—any overlapping, I think is…

Uma: It conflicts. I guess, yeah, that’s a good way to…

Sean: Does that make sense?

Uma: Yeah, that does make sense. What do you think about the overlapping, do they still have a relationship similar to what we were looking at before, or how do you even compare their relationships with the overlapping?

Sean: Well, I think we are subtracting exponents here.

Uma: Mmmhmm. And then you’d be comparing from there?

[DW standing behind the pair]

Sean: So, when they’re end to end there’s 10 to 1. If I overlap them all the way down, like we were talking, then they’re 1 to 1. So, when they’re partially overlapped, then they’re not 10 to 1. That’s what…

Uma: Mmmhmm. What could it be?

Sean: How do you define that? What could it be?

*Interaction ends when DW joins the conversation.*

Building Significance?

This interaction involves a series of talk-turns between Uma and Sean, attempting to grapple with a difficult problem. Throughout the interaction, Sean makes statements of his thinking, which Uma agrees with, positioning his thinking as significant. After Sean states his idea about the overlapping intervals, Uma compliments his thinking saying, “it’s a good way to [think about the problem].” Throughout the interaction, Uma does not share her mathematical thinking, instead she continually questions Sean about his mathematical thinking, positioning his thinking as significant.

Building Communal Practices?

Uma starts the conversation after DW leaves (from the previous interaction), stating what she thinks but inviting Sean’s thinking. The pair then goes through a series of exchanges where Sean states his thinking, Uma agrees, then asks clarifying questions or invites Sean to continue explaining his mathematical thinking to her. Uma continually invites Sean to reason with her, which are met with long (up to 15 seconds) pauses and statements of Sean’s thinking. After they both agree that they are at a standstill, Sean explains his thinking to Uma, asking for clarification along the way.

Interaction 6: Session 3 (31:00 – 31:54)

DW: That’s an interesting—I like that, that’s an interesting representation. Your thinking for that, that’s neat.

Uma: [laughs]

DW: So, I guess now I’m curious, if you jumped over to the hypothetical model, which, if you look at question 4 part b is repeat of b, c and d with the hypothetical model. I’m curious, because that was really interesting, I’m curious to know if something similar happens in this hypothetical model. Right, so you are basically saying that because these overlap they do not represent 1/10 of the amount of time…

Sean: right.

Uma: Mmmhmm.

DW: … because if they were completely overlapped, they would represent the exact same amount of time. So, there’s like somewhere in between.

Sean: there is.

DW: Ok, so I’m curious to know if that same, like, does that happen over here too, in the hypothetical model?

*Interaction ends with DW leaving.*

Building Significance?

This interaction begins with DW complimenting Sean’s thinking, positioning his thinking as mathematically significant. Specifically, DW’s description of Sean’s thinking as “neat” is quite a substantial compliment, as it implies that his thinking is unique.

Building Communal Practices?

DW poses a clarifying question to the pair, to which both Uma and Sean agree. DW then rephrases Sean’s idea about completely overlapping intervals, to which Sean agrees. DW questions both members of the group equally, despite having elevated Sean’s thinking at the outset of the interaction.

Interaction 7: Session 3 (31:58 – 34:10)

Uma: I would think you’d be in this similar situation, right?

Sean: I think so.

Uma: because if the only thing that changes between the hypothetical and ours is the interval—the amount of space and that relationship should still translate from one to the other.

Sean: Regardless of the base 2 or 10, the same phenomenon is still in place.

Uma: Mmmhmm.

Sean: So, here we have—this is how many years ago? So it’s 2 to the 6th, which is like 64, I think…

Uma: Mmmhmm

Sean: … and 2 to the 4th is 16? But 2 to the 2 is 4. So, they are, um, I mean that’s 12 years. What is that? I can’t do math. Is that 48 years? So, there’s 4 to 1 between those segments. But if we go and overlap them, then we’ve got 32 years here. And um 8 years here, which is 24 years, so 24 to 12, which is 2 to 1.

Uma: Yeah.

Sean: So, the same thing is happening.

Uma: Yeah, you’re changing that.

Sean: And, here’s kind of worth noting, I guess, is that when they’re end to end it’s 4 to 1, but if we overlap by halfway then it’s 2 to 1.

Uma: Yeah, cause you’re cutting it in half.

Sean: So, then if we overlap by ¾, I don’t know, then it’s, it’s not—What is that going to be? I don’t know what that’s going to be. So, what are the prompts? So, saying kind of…

*Interaction ends with both students working on their calculators.*

Building Significance?

This interaction primarily encompasses Sean thinking through the question DW posed to the group at the end of the previous interaction. Sean makes a series of statements about his thinking, to which Uma agrees, positioning Sean’s thinking as significant. The single time Uma makes a statement about the difference between the hypothetical model and their model Sean does not acknowledge her reasoning. Instead, he makes another statement.

Building Communal Practices?

At the beginning of this interaction, Uma states her thinking but poses an open invitation for Sean to describe his thinking. Her invitation is met by multiple statements from Sean, where he shares his mathematical thinking, until he asks for Uma to confirm his mathematical calculations. It isn’t until the end of the interaction, when Sean is unsure of how to continue with his line of thinking, that he invites Uma to move forward through the prompts.  
  
Sean also makes the statement, “I can’t do math”, which seems like he is seeking comfort.

Interaction 8: Session 3 (35:00 – 38:55)

Uma: Um, yeah, [re-reads prompt]. So, that’s kind of what you were doing with this part, right, when you cut it in half?

Sean: Well no, it wasn’t quite there. What I was thinking was that we would overlap by a half and then overlap by ¾, but I think now we’re talking about like this segment and that is three times the length.

Uma: Mmmhmm. And how does that compare? It says [reads prompt]. So, if you have like 2 [reads prompt]. So, if your length here is 2, then if you were to look at 4, is that what they are thinking?

Sean: Yeah, just a—ok, well I was looking at a length of 1, so just between 1 and 2 and 3 units. So, I went from 1 to 2, which is a difference of 2 years and these three units is 28 years? Right, 32 minus 4.

Uma: Mmmhmm. So, and then what is the amount of time between? We got…

[17 seconds]

Sean: So, let’s go with 5.05 times 10 to the 8th. What is this, the domestication of fire to the Ordovician…?

Uma: Domestication of fire is here.

Sean: The other one I don’t see… [5 seconds] Which I guess equals…

Uma: It says “Trilobites, corals, and shelled animals” [reading the timeline/prompt]

Sean: Ok, so right about 29, Trilobites?

Uma: Yeah, compared to the Miocene, which is the apes, monkeys and… The Triassic… cone bearing trees…That the first segment is roughly three times longer than the second segment [repeating the prompt].

*Interaction ends when DW poses a question and enters the conversation.*

Building Significance?

Uma poses a question to Sean about the interpretation of the next prompt, asking if it is similar to the idea he recently voiced. She notably asks him if this is similar to “what you were doing,” elevating Sean’s idea. Numerous times, Uma states her thinking or interpretation, and then invites Sean to voice his ideas, which positions Sean’s thinking (and his thinking of her thinking) as mathematically significant.

Building Communal Practices?

Uma invites Sean to clarify if the prompt is asking a similar question to the idea he recently voiced. This is the first instance where Sean genuinely explains to Uma why he doesn’t think his idea fits this scenario, rather than making a closed statement (e.g., “No, I don’t think so.”). Uma then talks through her interpretation of the prompt and then invites Sean to describe his interpretation. Sean then explains to Uma his thinking, and connects his ideas to the idea she voiced.

Interaction 9: Session 3 (38:55 – 41:30)

DW: So, what did you guys think about for the hypothetical model, with part c?

Sean: We think that the same, all that’s really changed is the base from base 10 to 2, so the same phenomenon, I think, that kind of sliding that I was talking about, still applies.

[5 seconds]

Sean: We didn’t really investigate that any further.

Uma: Yeah, no, I kind of, we kind of established previously that the real difference between our model and their model was the base. So, if that is all that’s changing, technically---

DW: Is that all that’s changing?

Uma: I think it was like the biggest.

DW: What about within, say 22 and 23 in the hypothetical model, like 22.17 or something like that. What would that, what does the space between those two marked things look like in the hypothetical model compared to the space between your, like your 10 to the 4th and 10 to the 5th mean?

Uma: Cause ours are 10 times the amount of space.

DW: Yeah, but when you say that, are you talking about these nice time points or are you talking about just in general?

Uma: I think just, in general (?)

DW: OK

Uma: From between this point and this point…

DW: But what about within those points though? How do you account for changes in time between 10 to the 9th and 10 to the 8th, and is that similar or different from the way this student accounted for time between say, 24 and 25? Or 25 and 26? Does that question make sense?

Uma: Mmhmm.

DW: OK

*Interaction ends when DW leaves.*

Building Significance?

DW initiates the interaction by inviting the group to share their thinking about how their model compares to the hypothetical student’s model. Sean responds with a statement, that Uma revoices. Both Sean and Uma interchange between “I” and “We” when referencing the thinking, so it is unclear whose thinking is being elevated. The original idea that the primary difference between the two models is the base was Sean’s. In that sense, by adopting that idea as the way of thinking within the pair, Sean’s thinking is being positioned as significant (but here the idea has been accepted by both students, and is functioning as if shared).

DW begins the interaction with an invitation to share thinking, which evolved into repeatedly asking questions (seemingly directed towards Uma). It is possible that, over time, Uma experienced DW’s persistence to mean that her explanations were insufficient to answer the questions, which could have worked to position Uma’s thinking as less-than-significant.

Building Communal Practices?

DW repeatedly asking questions (seemingly directed to Uma), on the one hand, gives Uma space to share her thinking. However, on the other hand, DW’s persistence could have diverged into discomfort for Uma.

# Interaction 10: Session 3 (42:21 – 44:43)

DW: Did you guys talk about it?

Uma: No, still trying.

DW: OK

Sean: I mean, I think, so in between those, just to use that example of 2 to the 22.5 and then on here, like I said, this model kind of morphed over time, but I can see there at some point I was thinking very similarly. I’ve got 10 to the 6.5 written on here.

DW: Right.

Sean: So, I think the same…

DW: I recall your 10 to the 6.5 because I asked you a question about it.

Sean: OK

DW: …and you were, it was the very beginning of the last session, and you were saying that you could switch your model to that representation, but that you had most of your other dates written in true scientific notation, so you were going to leave them in scientific notation

Sean: OK

DW: So, that’s where you 10 to the 6.5… so, I agree that you guys did think…

Uma: … about that…

DW: … in the same way, as this one [the hypothetical model], just in with a different base. That one being 2, this one being 10. But where you started, where these ones are coming from, those are not exponents, right? That’s not 10 to the 5.7, that’s 5.7 times 10 to the something.

Sean: Right.

Uma: Mmmhmm.

DW: So, in the scientific notation format, what’s happening within the segments compared to what’s happening between segments?

Sean: So, I think initiailly we starred with, even though our segments weren’t increasing at a linear rate, within them we had kind of a linear scale…

Uma: Yes.

Sean: …between them. That’s where it…

Uma: Yeah.

DW: So, it’s kind of a combo?

Sean: Kind of a hybrid.

DW: So, I’m curious in 4a. Can you talk about 4a together?

*Interaction ends with DW walking away.*

Building Significance?

Words

Building Communal Practices?

Words

Interaction 1: Session 5 (5:30 – 6:35)

*This session is different from the others in that Frank has now joined the pairing of Sean and Uma. Also in this session (beginning in session 3) we made the decision to intentionally assign pronouns of ‘she/her/hers’ when referencing the hypothetical student’s model).*

Frank: I was going with 5.1, roughly.

Sean: Uh, yeah. I was going to say 6 but I didn’t look at the scale. I was looking at 1 unit passed 5. Now I don’t know exactly what that- is that?

Frank: That’s segment on timeline- So hers was 2 to the t, so would it be 2 to the 5.1 then? [3 seconds] Because each segment was a power of 2.

Sean: It was

Uma: Yeah [leaning over to see Frank’s paper]

DW: Yeah so this segment was just the language that you guys, that we kind of collectively came up with. And it reflected that numbering system along, along her timeline. So I think it does—

Sean: OK

Frank: --represent that.

DW: And then you guys used segment to talk about the space between powers of 10.

Sean: Right.

DW: For your models. So that’s why I asked her to label it as ‘segment.’

*This interaction ends when DW leaves. The group was collaborating to understand the language being used in the prompt.*

Building Significance?

Sean positions Frank’s thinking as significant in this interaction by agreeing with Frank’s statement, and then saving face to justify why Sean’s own original thought was possibly incorrect. In doing so, Sean verifies Frank’s thinking as correct with “I was looking at 1 unit passed 5.”

Building Communal Practices?

In this interaction there is some back and forth between Sean and Frank. Frank begins by stating his thinking and then presents an argument about how he interprets the meaning of his first result. Sean’s response to Frank’s initial statement serves three purposes: (1) Sean agrees with Frank, (2) Sean makes a contribution to save face about why he originally misread the graph, which also works to verify/confirm Frank’s original thinking, and (3) Sean asks a question, which invites others to think more about the rest of the prompt. Frank accepts Sean’s invitation and makes an argument about how to interpret his response (the 5.1). Sean and Uma agree with Frank’s argument, but are unable to continue the conversation when DW clarifies what the prompt is asking.

Interaction 2: Session 5 (6:35 – 8:24)

Sean: So we’re getting a segment of 2 to the 5.1 corresponds to 35 years ago. [16 seconds. All three are looking on their own papers]

Frank: So yeah, 2 to the 5.1…

Sean: 3 point, what 3—

Frank: I got 3.4 for 10.

Sean: [Looking on Frank’s paper] 10 is one segment on the bottom.

Frank: Yeah [all students still looking on their own papers] are ours different?

Sean: No, it could be that I’m just not reading it correctly. So I’m just following that straight up [Frank is looking at Sean’s paper]. I think it’s well- I’ve written on it so.

Frank: [repositions his paper for Sean to see] I wonder if our graphs are different?

Sean: It’s very clearly—

Frank: Or maybe we’re estimating. Maybe it’s like right in there

Sean: Yeah, that’s the difference I think. So 3 point—

Frank: 3 point 3?

Sean: Yep.

Frank: [Looking to Uma] What did you come up with [laughing] for 10?

Uma: So it’s like right there isn’t it [pointing on her paper]?

Frank: Yeah.

Uma: What did you say? You said, 3 point—

Frank: 3 point 3 roughly because each line is point 2. So I said it was kind of half way between 2 and 4 – or point 2, point 4.

Uma: Yeah [nodding]

*The interaction concludes with all three students working independently for about 35 seconds. This interaction had two distinct instances: 1) Frank discussing with Sean, and 2) Frank discussing with Uma.*

Building Significance?

There are two distinct instances in this interaction: first, where Frank is discussing with Sean, and second, when Frank is discussing with Uma.

In the first portion of the interaction, Frank and Sean do not really position either’s thinking as mathematically significant. They both agree on the corresponding value from the graph associated with a horizontal coordinate of 35, and then discuss what corresponds with a horizontal coordinate of 10. See Building Communal Practices about how the nature of this conversation worked to establish a collaborative environment.

Frank initiates the second portion of this interaction by inviting Uma to share her thoughts about the corresponding vertical component for a horizontal coordinate of 10. In this portion, Frank presents an argument for his claim that the corresponding value with 10 is 3.3 based on the scales of the provided graph. Uma agrees with Frank’s argument, which positions his thinking as mathematically significant.

It is interesting that Frank does not present the same argument when discussing with Sean.

Building Communal Practices?

During the first portion of this interaction, when Frank is discussing with Sean, Sean appears to have made a mistake in reading the provided graph for the prompt. In response, Frank offers Sean comfort twice by questioning whether their two graphs are the same. Offering comfort is an interesting contrast between how Frank interacts with Uma, where he presents an argument for his claim that 3.3 is paired with 10. Sean and Frank can be seen working on the same graph to arrive at a joint conclusion that 3.3 is paired with 10. The two reach consensus without providing an argument for their claim – The two are working together to produce their response.

After Sean and Frank arrive at their response, Frank invites Uma to share her thoughts. This was an open invitation for Uma to collaborate. Uma responds to Frank’s invitation with a set of questions that lead to Frank presenting an argument for his claim that 3.3 is paired with 10 on the graph. Arguing is an open way of establishing a collaborative environment because it enables others to interact with one’s mathematical thinking and reasoning. Uma is receptive to Frank’s argument, and agrees with his claim – consensus is reached.

Interaction 3: Session 5 (9:36 – 12:32)

Sean: So you have a question about your uh 8.4, is that the rules of exponents or is that[?]

Frank: Yep. Yeah, don’t they add?

Sean: I don’t think so, because 2 squared plus 2 cubed is 4 plus 8, right?

Frank: Which is—

Sean: But 2 to the 5th is going to be, 16, 32.

Frank: This is…

Sean: 2 cubed. I just did that as an example, right.

Frank: Yeah, no you’re good. [thinking aloud about the calculations he’s doing]. Maybe it’s not. Oh, maybe it’s multiplication not addition.

Uma: Yeah, I’m trying to remember. I can’t remember what---

Frank: Well either way, right.

Sean: Yeah

Frank: We started at 5.1 and 3.3. So if we summed- Well, so even if we did that, we’re taking 5.1 which is our output for A, right? And then we’re adding our 3.3—

Sean: Oh, yep. Now I’m, now I’m with you.

Frank: So it’s now 2 to that

Sean: I’m on board with that.

Uma: M hm.

Frank: But you are probably right there.

Sean: No that’s…

Frank: So what you did… so 10, 20, 30, 40, so 340 years.

Sean: Yep. That’s what I get too.

Frank: [reading the prompt] What relationship do we notice about A, B, C, f(A), f(B), and/or f(A) + f(B)?

Sean: Now moving along our timeline, if we multiply that 10 times 35 gives us 350—

Frank: 350 [simultaneously with Sean]

Uma: M hm

Sean: So that’s about the 340, right.

Frank: Yeah

Sean: So we can assume there is some error in our measurement probably.

Frank: Um, that’s the idea of multiplication with bases. So 2 to the 5.1 times 2 to the 3.3, is the same as 2 to the 5.1 + 3.3—

Sean: There we go

Frank: Yep, so that’s what we’re getting. [To Uma] Does that make sense?

Uma: M hm

Frank: So we did 10 times 35 is 350, which is kind of close to where we were at—[340]

Uma: Yeah, we were at 340 [simultaneously with Frank]

Frank: So our estimation might have been a little bit off here.

Uma: Yeah.

Frank: Um so what we’re doing is multiplying our bases here and we’re adding – because they’re causing our exponents to add that’s

Uma: M hm. Yeah, so 3 and 5, yeah

Frank: Yep.

*Then, the session concludes with a whole group summarization of progress on the prompts.*

Building Significance?

At

Building Communal Practices?

As

Interaction 4: Session 5 (12:33 – 16:51)

DW: So I didn’t ask you to multiply anything. So I’m curious to know where this multiplication is coming from.

Frank: Um, a comparison of A, B and C. C is equal to A times B.

DW: Ok [4 second pause]

Sean: What I noticed, I guess, I think how we got there was just that this quantity – the uh sum of the first two f(A) and f(B) puts us up here at f(A) + [f(B)], which corresponds to 340 years. We noticed that was pretty close to the multiplication of the first two dates we got.

DW: Ok

Sean: And I think that should be precisely that, but there’s some error in our measurement or something.

DW: Why do you think it should be precisely?

Frank: Because that’s the rules of exponents.

DW: I guess that’s my question. Where—I don’t see—Continue with that. What do you mean, “because that’s the rules”? Why are these rules in effect here? Why?

Frank: Um, well her, the other person’s chart, her timeline was 2 to the power of t. Then so each segment on her timeline would be a t – like this would be t, that would be the t-axis or whatever [labeling the vertical axis on the provided graph]. And then number of years ago would be, I don’t know, some other axis but, with these exponents [pointing to the t-axis] um you had us adding them together and so the rules of exponents when your adding exponents you’re multiplying the bases.

DW: Ok

Frank: So then the equivalent pieces on the x-axis to the y-axis, if the y’s getting added the x’s getting multiplied. So there’s a covariation going on there.

DW: Ok [5 second] You called these exponents a second ago. You named them t but then you just said they’re exponents.

Frank: Well the exponent is t in her situation.

DW: Ok, Ok. So you’re saying because these were exponents and you’re being asked essentially to add exponents, that’s how YOU [emphasizing Frank] knew it corresponded to multiplication. And that just happened to [turning attention to Sean] be—

Sean: just by observation of the quantities. I would say it was a guess, you know based in some experience but it was a guess.

DW: Sure. Ok. How about you [to Uma]?

Uma: I think I was more along his lines of adding the exponents together, once you remember what the rules are for exponents.

DW: Right. And you wrote it down differently from the other two. You actually wrote f(10) equals 2 to the 3.3.

Uma: M hm [matter-of-factly]

DW: Which is- I think is slightly different from what the way you all were writing it down. SO yeah, it’s interesting. Um. You guys are talking about exponents and properties of exponents, but I don’t really see um like where are exponents living here? [5 seconds] Or how would you generalize this relationship maybe? Between these two quantities, right? We’re claiming that this graph is the relationship between these two quantities. Can you come up with a general relationship?

Frank: Are you talking about the mapping from G to A?

Uma: [laughing]

Frank: You’re bringing back nightmares of logarithms

*Following this interaction, the group works independently for nearly 3 minutes. Sean suggests that making a table would be useful for him. Frank gets a resource/worksheet from a class he and Sean were taking.*

Building Significance?

At

Building Communal Practices?

As

Interaction 5: Session 5 (19:53 – 21:20)

Frank: So in this case our b is just one.

EBO: [17 seconds] So how do you know your b is one?

Frank: Um, I’m just spit-balling at this point but if this is our covariation as, as we move up this y-axis or t-axis, if we have A to the m plus n, A is 2 because that’s the way she [the hypothetical student] set it up, um then 2 to the 5.1 plus 3.3 like there, the values of that is equivalent to the values of m and n multiplied together. So we’re not including any other constant out in front, so we’re not including any stretching or shrinking of our graph. So that coefficient out in front needs to be a one.

*This interaction ends with the group continuing to work independently on their own papers. In the 2:30 that elapse between this interaction and the next, Frank can be heard asking Uma if she is ready for prompt 2 – she indicates she is – and the two wait for Sean. Sean explains that he is trying to finish something up.*

Building Significance?

At

Building Communal Practices?

As

Interaction 6: Session 5 (23:10 – 24:11)

Frank: Yeah, I think that if our numbers were right on in these spots, then we would get to 350.

Sean: Yeah that’s per any kind of um…

Frank: Any kind of thing, yeah. So like if we did 100 is roughly 6.6 or whatever…

Sean: Oh wait so that’s…

Frank: Which in a sense that makes sense because 10 times 10 is 100, 3.3 plus 3.3 would be 6.6 which gets us to that rough point there.

Sean: [nodding] True.

*The interaction ends with Frank deciding to change prompts.*

Building Significance?

At

Building Communal Practices?

As

Interaction 7: Session 5 (24:28 – 26:50)

Frank: Ok, I’m going to move on to part 2 with you [talking to Uma]. Maybe not. Oh God, what’d you get for your—

Sean: For 3.5?

Frank: Yeah [turning to Uma’s paper] what’d you get for-- ?

Uma: Well, that’s like right about there [pointing on her paper].

Frank: M hm, so what’d you get for your x-value?

Uma: It looks like it’s a little over. Maybe like 12.

Frank: Yeah, I was thinking that too. [reading the prompt aloud] What number corresponds to, M, call this M. Weird, M and N already? I feel like I already have an M and N.

Uma: [laughing]

Frank: 80 on 6.5, roughly.

Sean: M hm [affirming]

Frank: [Reading the prompt aloud] Predict the number of years ago, X, that corresponds to… So we have to do that… [getting quiet]… If we wanted to do that, we could just estimate, or just say 80 times 12.

Sean: 960, is that—

Frank: M hm. Which then if we went to 10, which is not there, it would be somewhere around here.

Uma: Yeah.

*Then, the session concludes with a whole group summarization of progress on the prompts.*

Building Significance?

At

Building Communal Practices?

As

Interaction 8: Session 5 (26:50 – 30:21)

DW: Does 960 seem like a feasible?

Sean: It does to me because you’re getting, getting pretty linear out there

Frank & Uma: M hm.

DW: Well you’re claiming that you know.

Frank: I’m not claiming anything. I’m estimating, guestimating.

DW: Let me rephrase. You used this idea that the student’s hypothetical model that she labeled it as 2 to the t years ago and you used that specifically – And Uma, you used that to talk about the pattern that you saw in problem 1. So you’ve already used that 2 to the t before. Can you not use it now to—

Frank: We kind of created a rule

DW: --To see if that would make sense? What do you mean you’ve kind of created a rule?

Frank: Well it’s like a covariation, like because or correlation, whichever, correspondence, um—but we were able to take if we’re given say two segment lengths, distances, and we’re told to add them then we can approximate well, if our system is correct, we could approximate the value that would be plotted on this equation. We could find the number of years ago based on that.

DW: So if I gave you two segment lengths, asked you to add them together, you have a system for how you could find what the sum corresponds to. Is that what you’re saying?

Frank: Yeah.

DW: Care to share that system?

Frank: A to the M plus Nnnn, or 2 to the M plus N corresponds to M times N. Like we are mapping from—

DW: 2 to the, I’m sorry can you say that again?

Frank: If the, if the operation for G is 2 to the M plus N, it will map to A where A’s value is M times N.

DW: Ok, Uma, who are G and A? [Asking because Frank is referring to a task completed in a course that Uma was not enrolled in]

Uma: I don’t know.

DW: Ok, so can we try to bring—Frank, I think I understand what you’re saying because I was there when we did this example in class but I’m curious to know how you are drawing on that in this context. I’m going to assume that there’s a closer distance between how you two are able to draw from that [Frank and Sean], but Uma she’s not in that class, so she hasn’t had the privilege. So try to put this into the same context, so I can then ask Uma again who are A and G and she might understand who they are.

Frank: Ok

Sean: Yeah

*The interaction ends when DW leaves the conversation.*

Building Significance?

At

Building Communal Practices?

As

Interaction 9: Session 5 (30:35 – 36:45)

Sean: Well A is our linear scale and that’s across the bottom right? The x-axis?

Frank: Yeah, yep.

Sean: Then G is your vertical axis, a geometric function.

Frank: [35 seconds] Um, so yeah, basically G, we could relabel these this is G and this is A.

Uma: Ok

Frank: Um, and those are the outputs. But, A, like each of them has its own like function. Um, like its own operation going on.

Uma: Ok

Frank: Um, since like the operation for A is we take a value and multiply it by another value, it will pop out somewhere on here. So like 100 times 10—maybe not that. Maybe 10 times 50 would give us 500.

Uma: Ok

Frank: so it’s taking these values and we’re mapping to there. Um, so what we’re saying is we take these two values which are on here, take their x components and they will map a multiplication to this x component. Whereas, we take the y components, which are the function M plus N when we add those two together to get our y component of that piece. It’s kind of like a, it’s kind of like um like mapping inputs to outputs, but instead you’re mapping a set of points to another point [looking to Sean]

Sean: It’s a correspondence, right. Because you can’t use M and N to calculate the years ago, only use that the two kind of correspond, right. [inaudible] So if I have 2 to the 1 and 2 to the 2, I can’t use that 1 and 2 to calculate anything along here.

Frank: correct.

Sean: Is that alright?

Frank: Yeah. Isn’t that covariation though? I mean they can be done both ways – [reading from the class handout] corresponding numbers in G map to their corresponding sums. So in our situation we have the sum of two numbers in G map to their corresponding multiplication… product?

Sean: Sure.

Frank: Product, that’s the word, it’s right there! [laughing] So this is like the sum of two numbers in G map to the product of those numbers. And this is kind of like our x-axis and this is kind of our y-axis.

Uma: Oh ok.

Frank: In a sense.

Uma: In a sense, but you can’t look at it exactly like your y and your x.

Frank: Right, because I mean if we took, I mean it is true because if we took let’s say a G is 2, well 2 to the 2, maps to whatever which is something in here—so it’s going to map to a point, which makes sense. But if we say take multiple entries in G and we add them together, we can take that and find their multiplication on the other side in A, which corresponds to a point on our function. And this is f(t) equals 2 to the t. That’s not right. It’s f of something, let’s call it –

Sean: The curve

Frank: The curve, I like it. Ok

Sean: The curve is like G of A of m or something, right.

Frank: Yeahhh

Sean: Right, because G is a function of A.

Frank: Yeahhh

Sean: Maybe? [laughing]

Frank: No you’re, yeah. A. Yeah, kind of.

Sean: And that’s kind of what I’m getting at, is it’s…

*DW enters to end this interaction and start a new one.*

Building Significance?

At

Building Communal Practices?

As

Interaction 10: Session 5 (36:45 – 6:35)

DW: So Uma, instead of me asking, ‘whose G’, and ‘whose A’, can I rephrase that question and say—I want to get more into the last part of question 1 here. So how do you know? I buy your argument for this properties of exponents thing, and I understand that it’s coming from the hypothetical student, she labeled her axis as 2 to the t years ago. And put t as her timeline, right, this number of segments was corresponding to t. Um, but how do you, how do you um, how would you describe the relationship between these two quantities? The segment on the timeline and years ago. I would like a description of that relationship for how those two quantities go together according to this model.   
  
*Frank and Sean are looking at their own paper, not making eye contact with DW. Uma is looking back and forth between them and DW. 27 seconds elapse.*

Frank: Is that kind of what you’re asking?

DW: I don’t know I can’t really see it.

Frank: Well like in the logarithm, the product of two numbers in G are equivalent to their corresponding sums.

DW: Ok, so what does that mean here? I want to know about these two quantities. Because you’re still talking about G’s and A’s, and I want to know about segment on the timeline or distance on the timeline and years ago or elapsed time.

Frank: The sum of two segments on the timeline are equivalent to the product—

Uma: Of the segments

Frank: of their corresponding years ago.

DW: Does that—so you said ‘of the segments’… And Sean, you’ve been relatively quiet.

Sean: Yeah, well I’ve got written down basically what you just stated, right. The sum corresponds to the product. So we’re probably going to be going back to it again, but if I raise 2 to whatever’s along here, then I get the corresponding value. So then 2 to the 3.5 or whatever was 10.

DW: Ok, then you guys seem to be—this is good—so you’re describing the relationship between these two quantities as how segments on the timeline is mapped to years ago. What would the other direction look like?

Sean: Ok, so if I have 2 to the x…

Frank: Well wouldn’t it just be that the product of two numbers of years ago map to the sum of two segments on the timeline?

DW: Yeah. I think so.

Uma: Yeah. Because it goes back---

DW: Yeah, because it goes the other direction because it’s going back and forth. Alright, so that’s—then is that how you guys think these two quantities are related to one another? That sums, sums over here [pointing to the t-axis] correspond to products-

Frank: -products- [simultaneous with DW]

DW: -- over there [pointing to the horizontal axis], and the other. It doesn’t matter which one comes first, like you could say products over here map to sums or sums map to products.

Uma: yeah, map to products [simultaneous with DW]

DW: The way you all reflected it was just here to there?

Frank & Uma: Yeah

DW: Ok, ok.

*Then, the session concludes with a whole group summarization of progress on the prompts.*

Building Significance?

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Building Communal Practices?

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